Radical Uncertainty in Subsurface Science

How do we make decisions?

Prof. Andrew Curtis

Radical Uncertainty in Subsurface Science

How do we make decisions?

Special credit to

Richard Arnold, Xuebin Zhao and Xin Zhang ***+ 1***

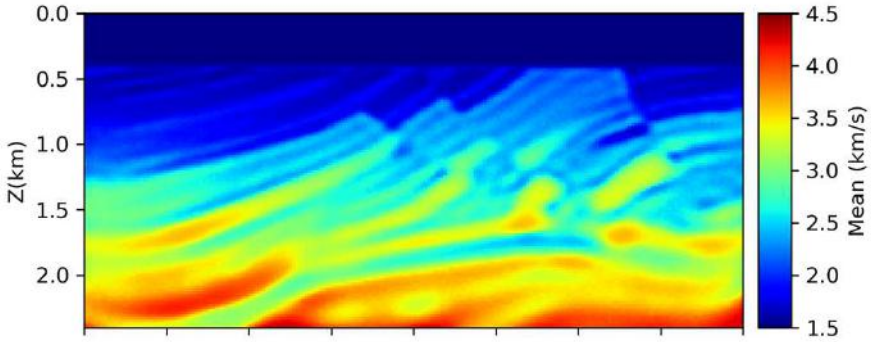
Interrogation Theory

Seek **low-dimensional** answers by interrogating models & data

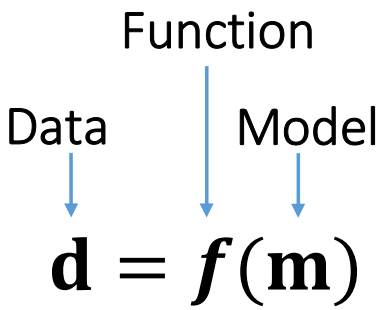
- What is the volume of a particular subsurface reservoir? → *X km^3*
- Has more than *1 Kt* of CO_2 escaped from the subsurface store? → *Yes/No*
- Which dynamic model best explains the *true* seismicity? → *Model 1,2,3…*

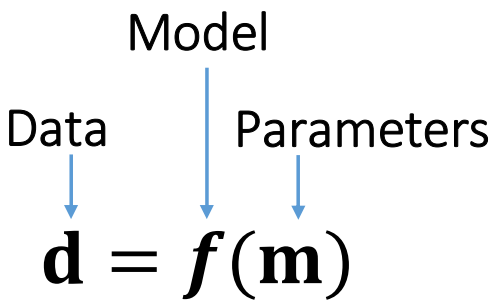
Answers often lie within **high dimensional spaces**

- Introduce Interrogation Theory
- 2D seismic imaging example
- 3D & 4D interrogation is now feasible
- *Radical* uncertainty: How do we make decisions?

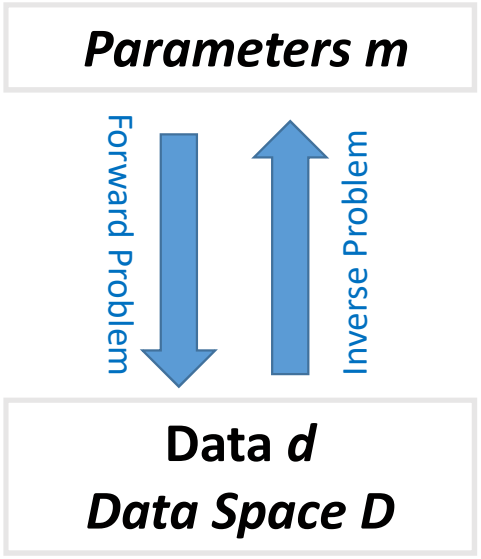


Inverse Theory and Interrogation Theory

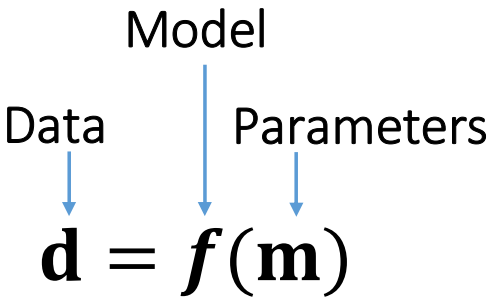




Inverse Theory and Interrogation Theory



Inverse Theory

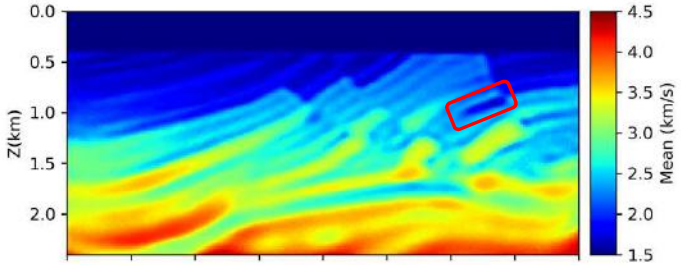


→ *Find parameter values that fit observed data best*

The parameter values are definitely wrong!

Inverse Theory and Interrogation Theory

Has more than *1 Kt* of CO_2 escaped from the store?



Parameters *m*

Forward Problem

Inverse Problem

Data *d*

Data Space D

➔ Bayes' theorem

$$p(\mathbf{m} \mid \mathbf{d}_{obs}) = \frac{p(\mathbf{d}_{obs} \mid \mathbf{m})\,p(\mathbf{m})}{p(\mathbf{d}_{obs})}$$

likelihood

Prior pdf

evidence

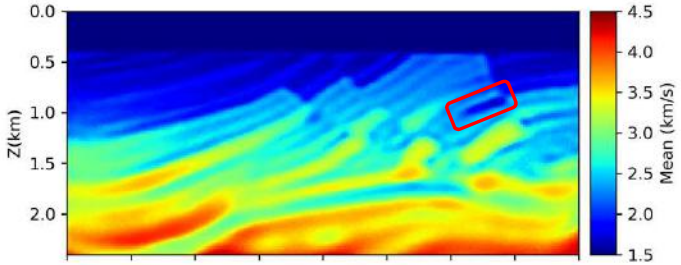
Posterior probability distribution function (pdf)

Inverse Theory

➔ Probability that **m** is true – given model assumptions & observed data $\mathbf{d}_{obs}$

Inverse Theory and Interrogation Theory

Has more than *1 Kt* of CO_2 escaped from the store?



Parameters *m*

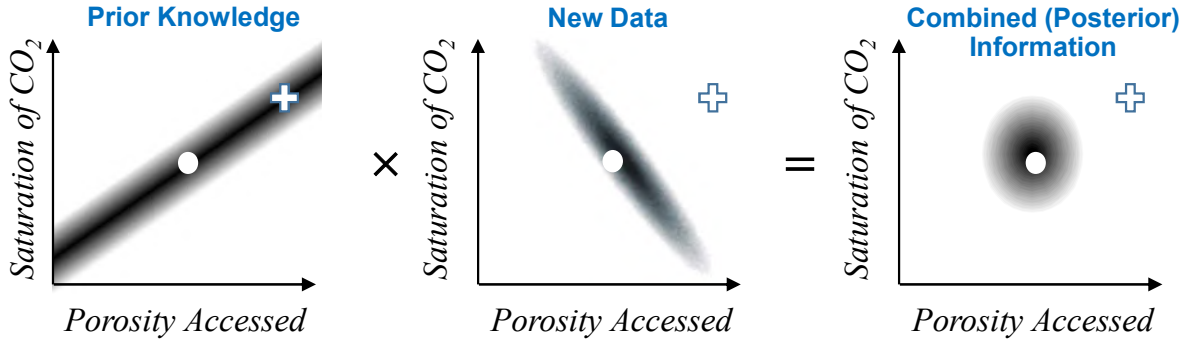
Forward Problem

Inverse Problem

Data *d*

Data Space D

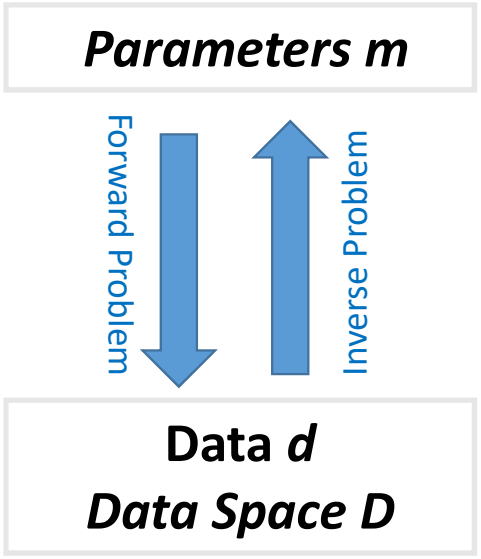
➔ Bayes' theorem



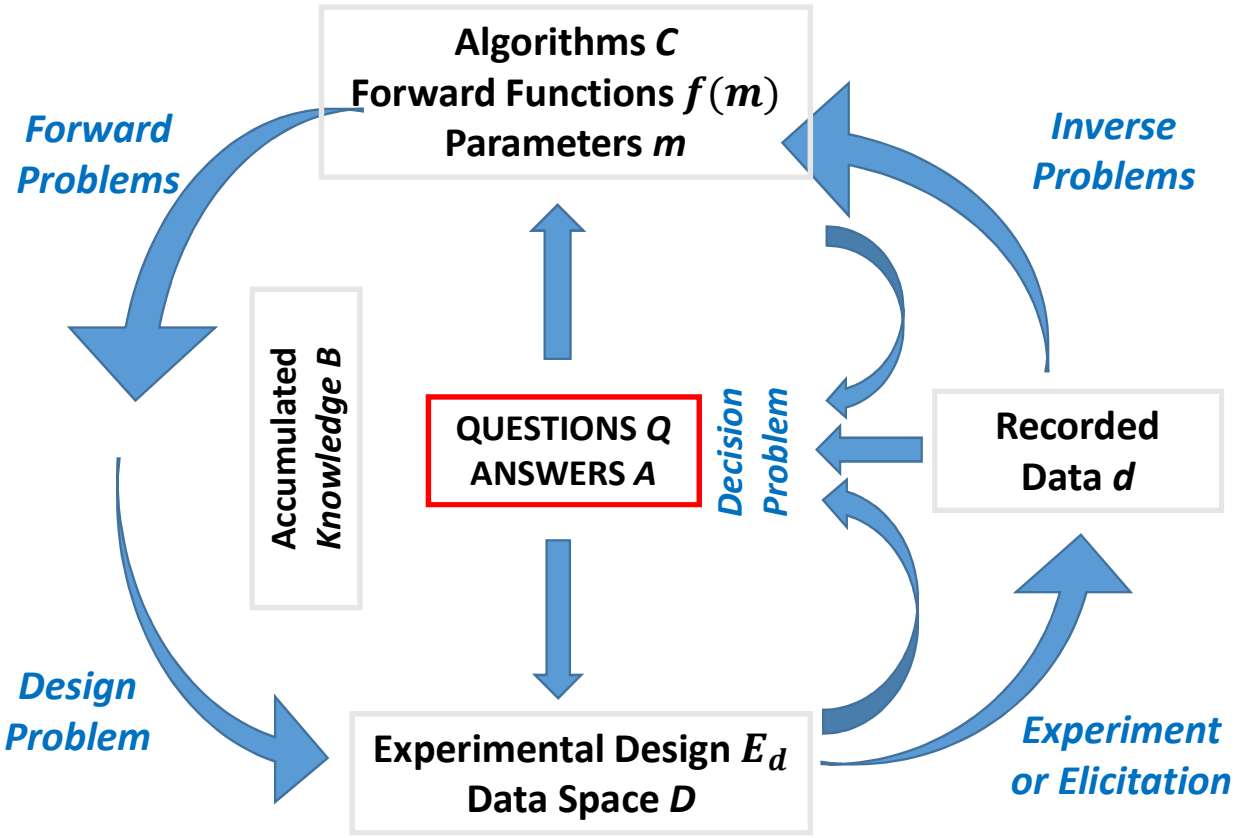
Given our question, which parameter values should we evaluate?

Inverse Theory

Inverse Theory and Interrogation Theory



Inverse Theory



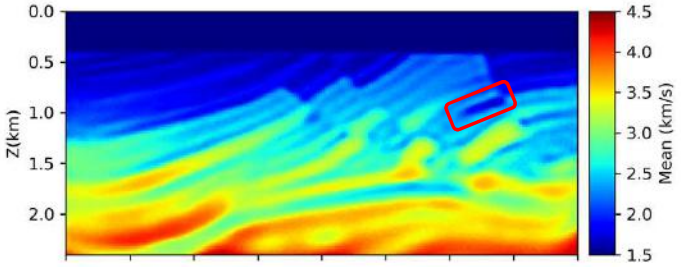
Interrogation Theory

Arnold & Curtis (2018) Geophys. J. Int.

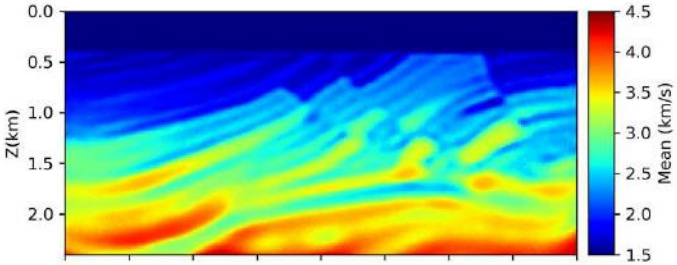
Synthetic Example

Decision: If reservoir volume > V, consider it as a CO_2 storage asset

Q: What is the volume of this reservoir?



What is the size of a reservoir?



- **Parametrization**: θ_m= seismic velocity V(**x**)
- **Data**: Seismic full waveforms (data y_d, design d)
- **Modelling**: Finite difference modelling (model m: $\theta_m \rightarrow y_d$)

- Define target function $t = T(\theta_m|m, Q)$:
  transforms any velocity structure θ_m into corresponding reservoir volume
- Define Utility: $U(a|t) = -(a-t)^2$ Defines the net benefits of any answer

Arnold & Curtis (2018), Geophys. J. Int.
Zhang & Curtis (2022), Geophys. J. Int.

What is the size of a reservoir?

- **Parametrization**: θ_m= seismic velocity V(**x**)
- **Data**: Seismic full waveforms (data y_d, design d)
- **Modelling**: Finite difference modelling (model m: $\theta_m \rightarrow y_d$)

- **Optimal Answer** is then:

$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m) p(\theta_m, m|y_d, d)\, d\theta_m$$

Arnold & Curtis (2018), Geophys. J. Int.
Zhang & Curtis (2022), Geophys. J. Int.

What is the size of a reservoir?

- **Parametrization**: θ_m= seismic velocity V(**x**)
- **Data**: Seismic full waveforms (data y_d, design d)
- **Modelling**: Finite difference modelling (model m: $\theta_m \rightarrow y_d$)

- **Optimal Answer** is then:

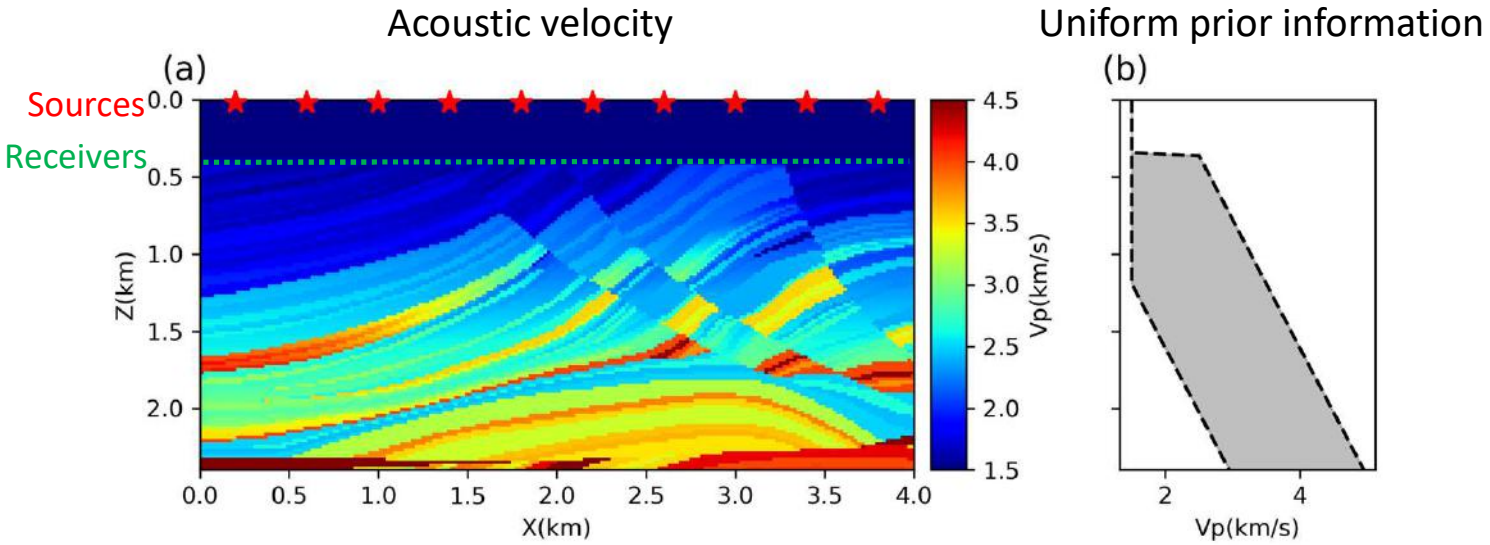
$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m) p(\theta_m, m|y_d, d)\, d\theta_m$$

Average over all possible models — Target — Bayesian Posterior pdf

Arnold & Curtis (2018), Geophys. J. Int.
Zhang & Curtis (2022), Geophys. J. Int.

Variational full-waveform inversion

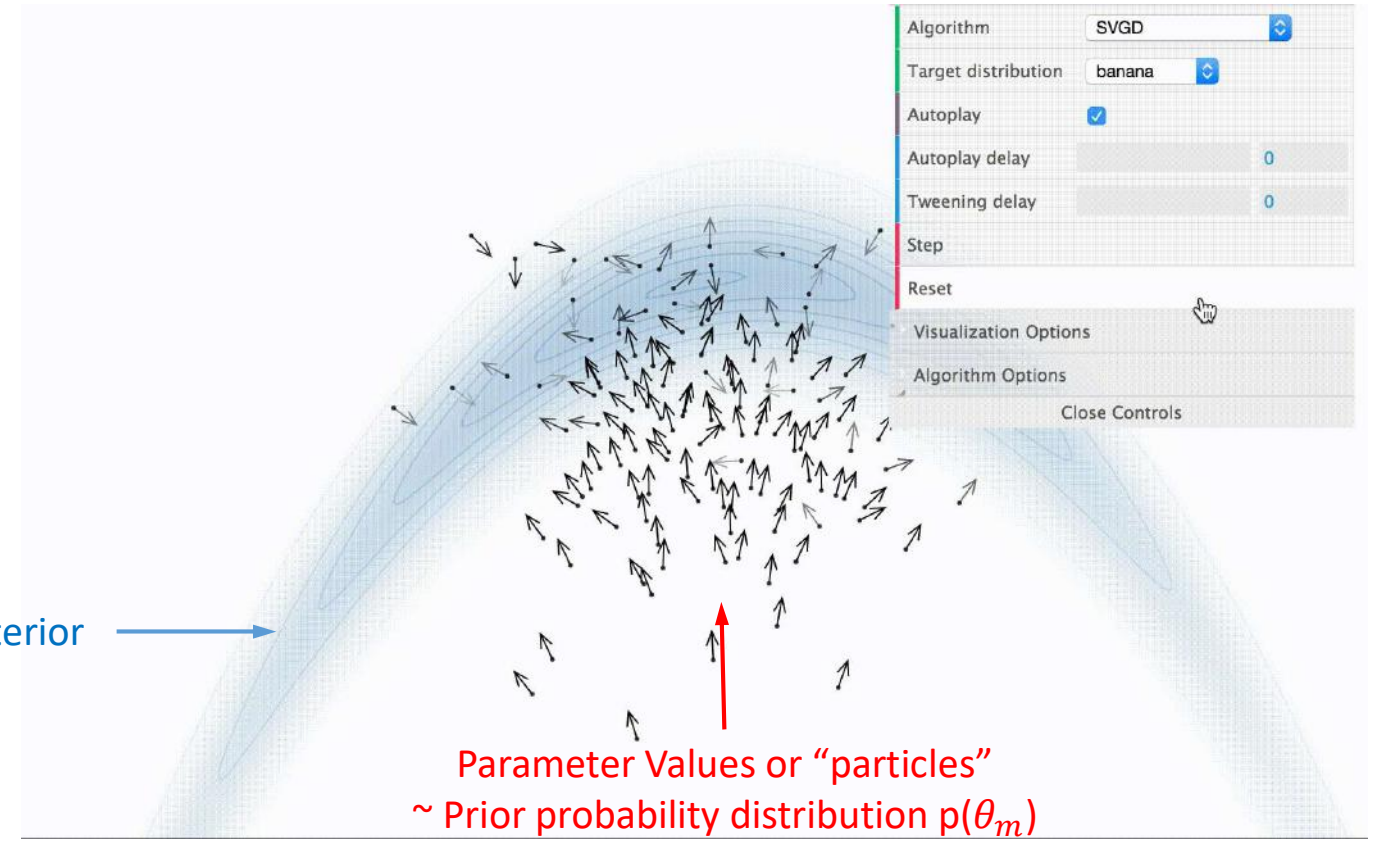
- Acoustic Marmousi model
- 10 shots, 200 receivers
- 200*120 free parameters
- Uniform prior
- Data simulated using 4 Hz Ricker wavelet



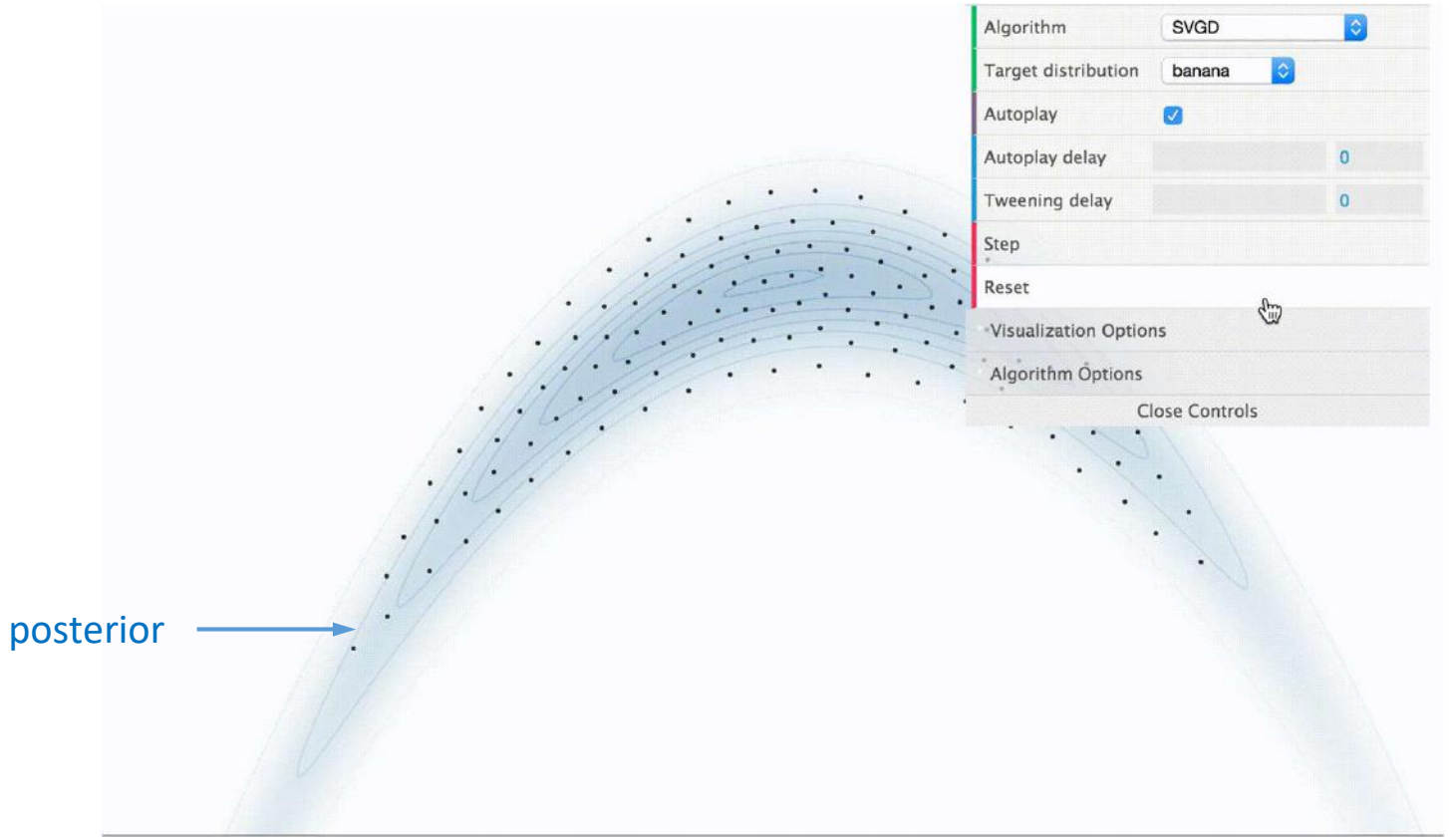
$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m)\, p(\theta_m, m|y_d, d)\, d\theta_m$$

Zhang & Curtis (2022), Geophys. J. Int.

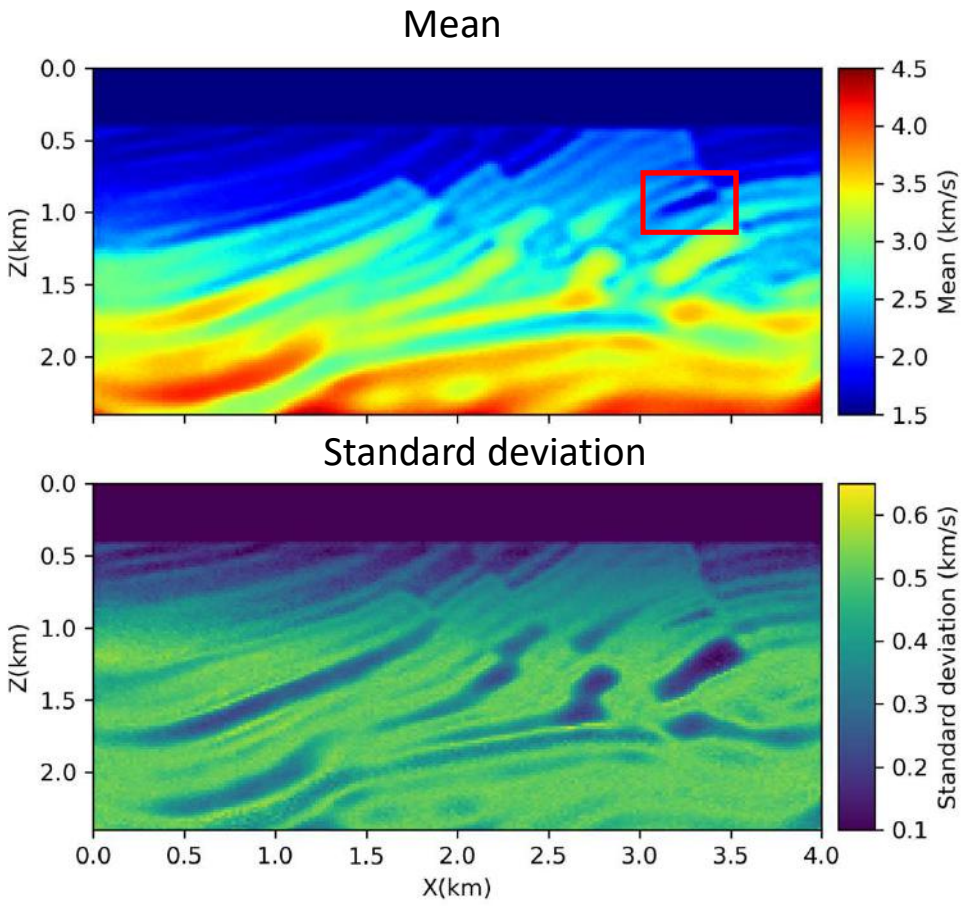
Stein variational gradient descent (SVGD)



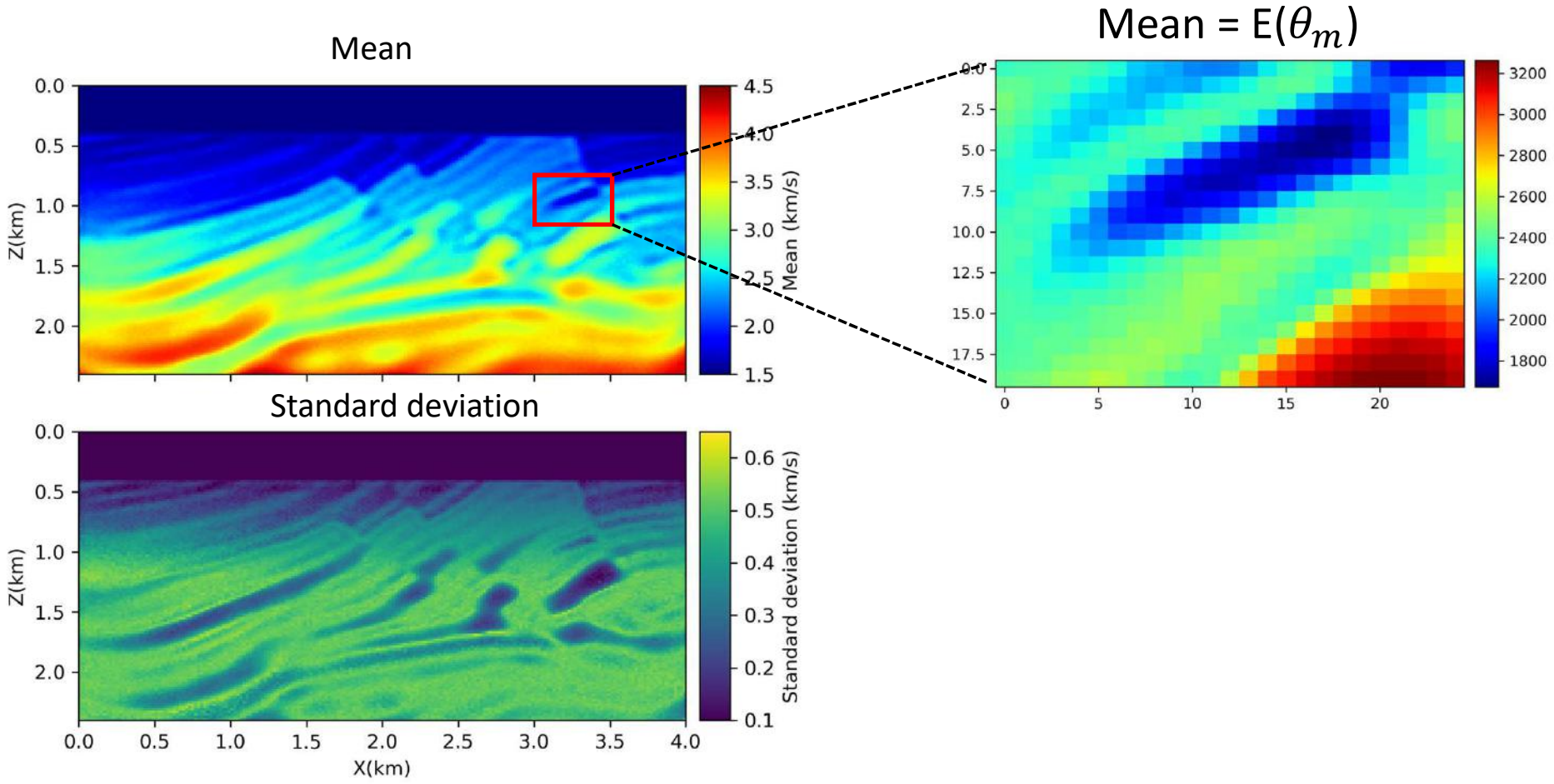
Stein variational gradient descent (SVGD)



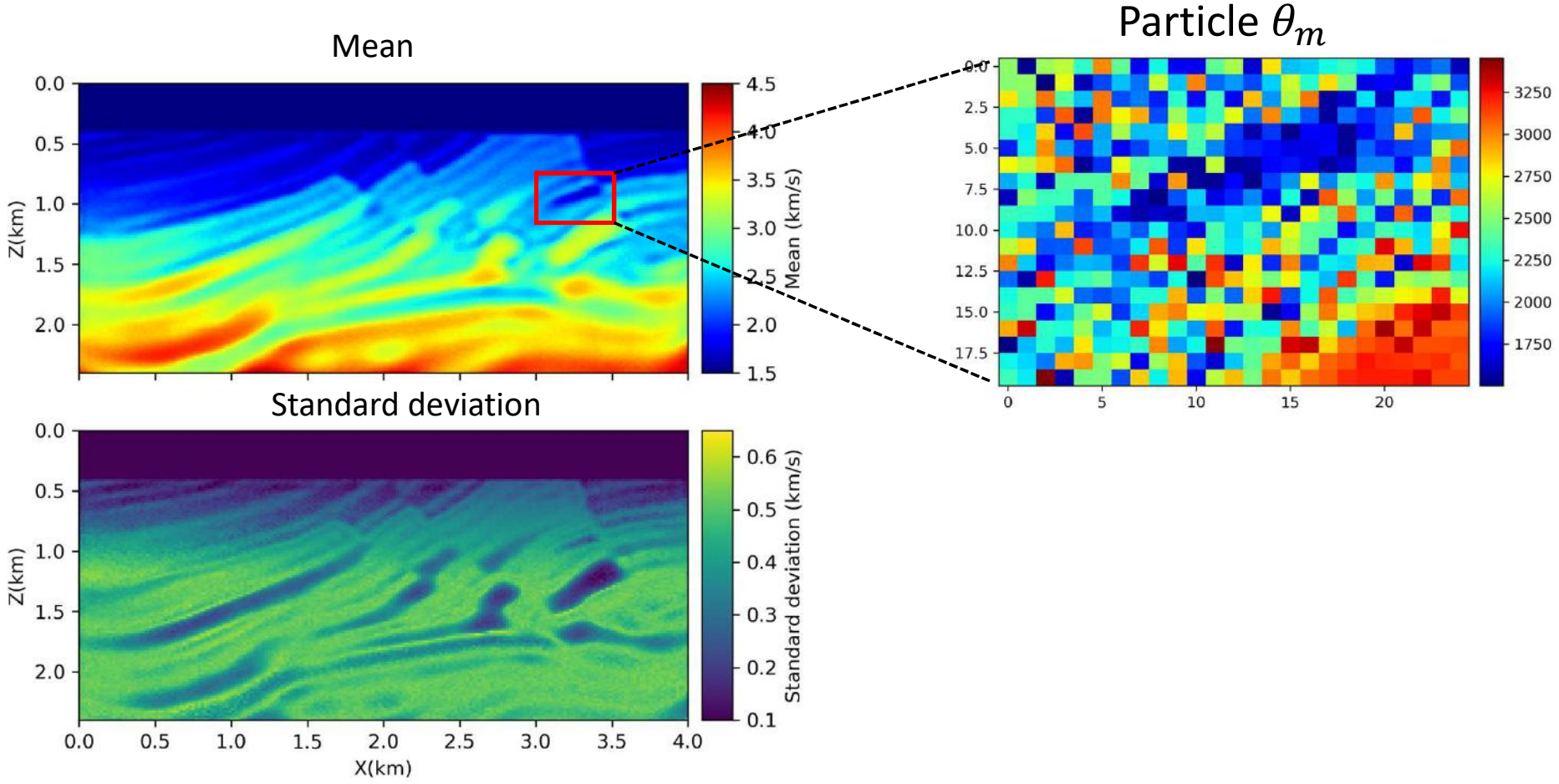
What is the size of a reservoir?



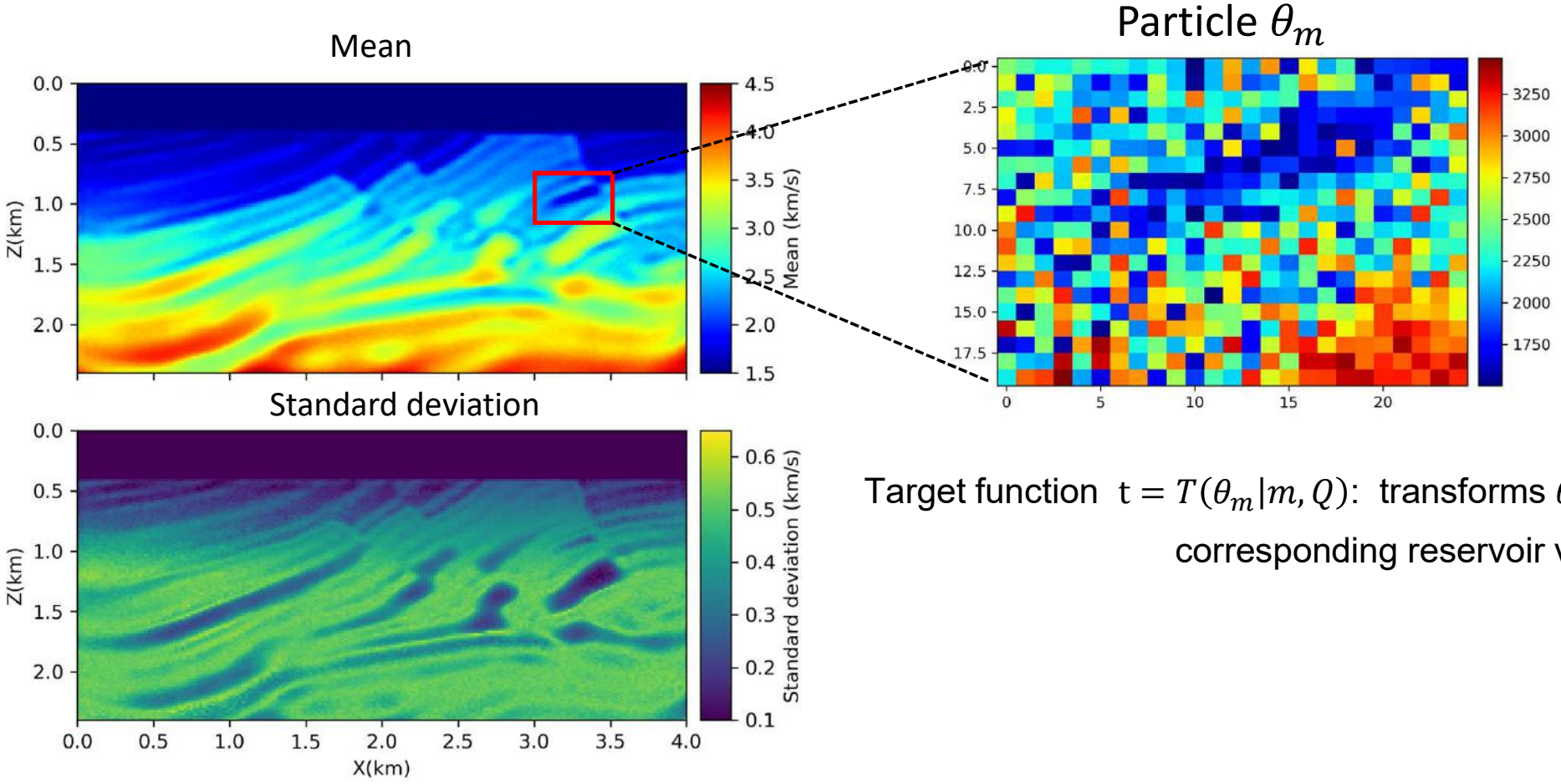
What is the size of a reservoir?



What is the size of a reservoir?

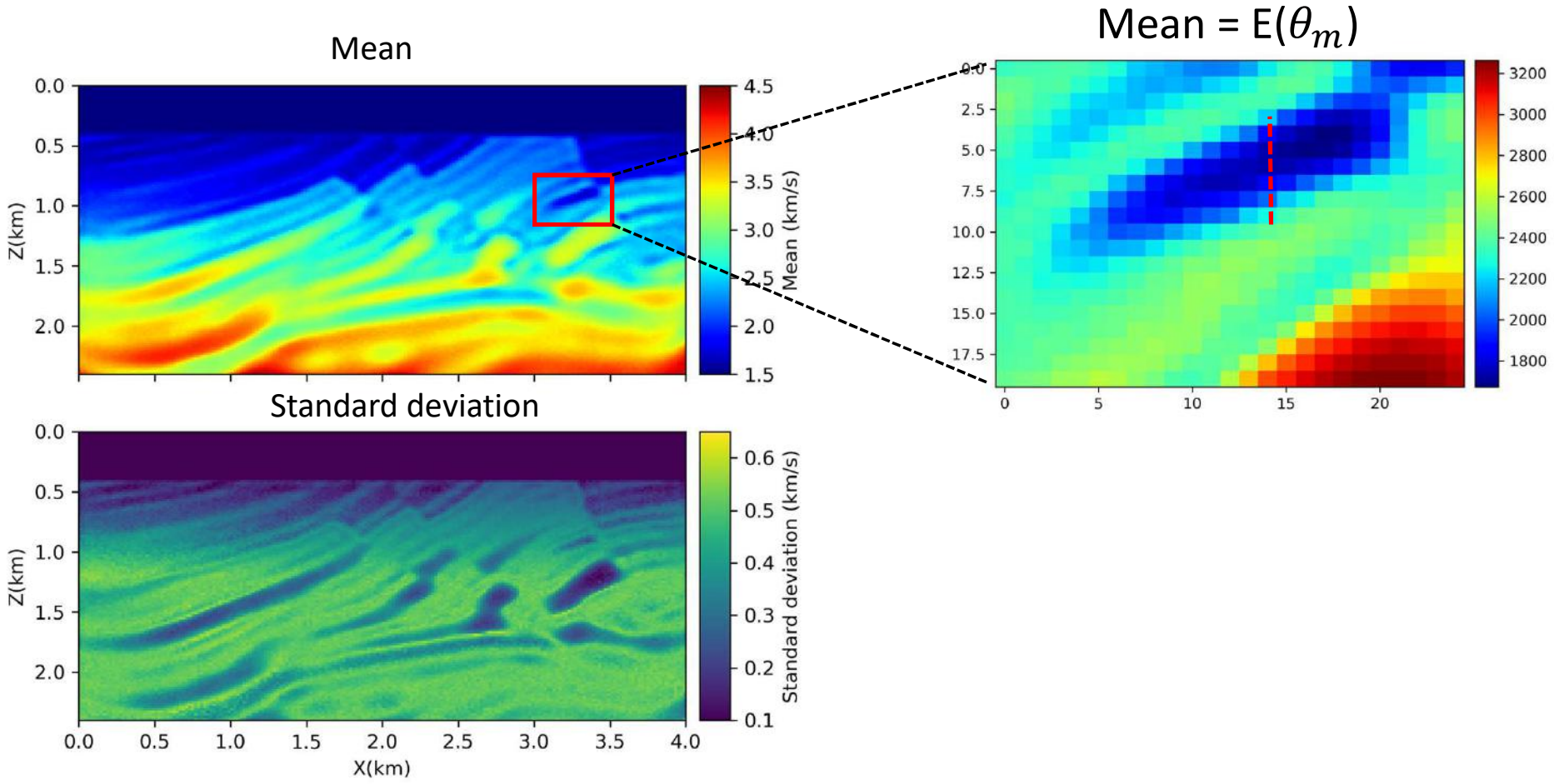


What is the size of a reservoir?

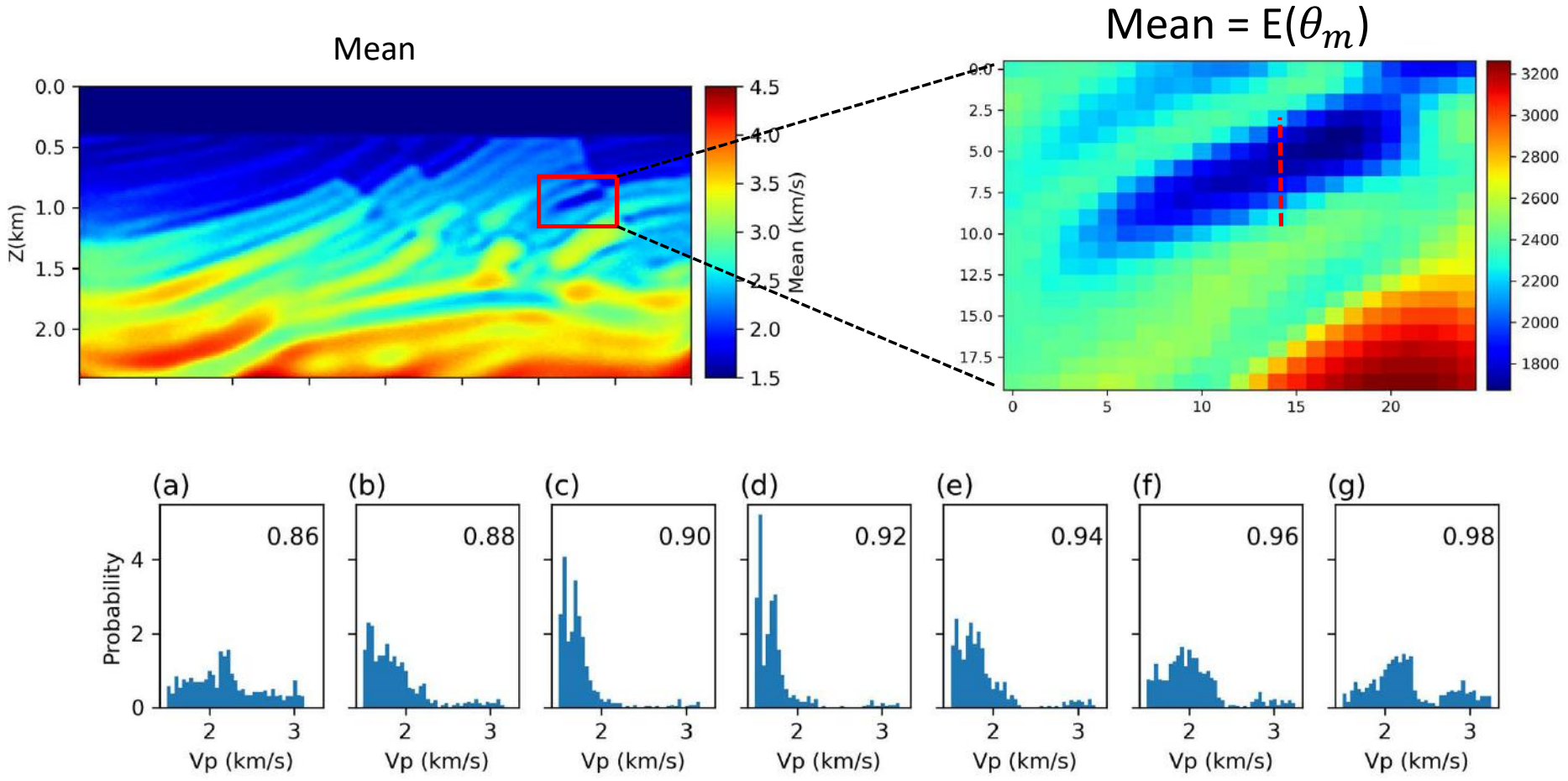


Target function $t = T(\theta_m|m, Q)$: transforms θ_m into corresponding reservoir volume

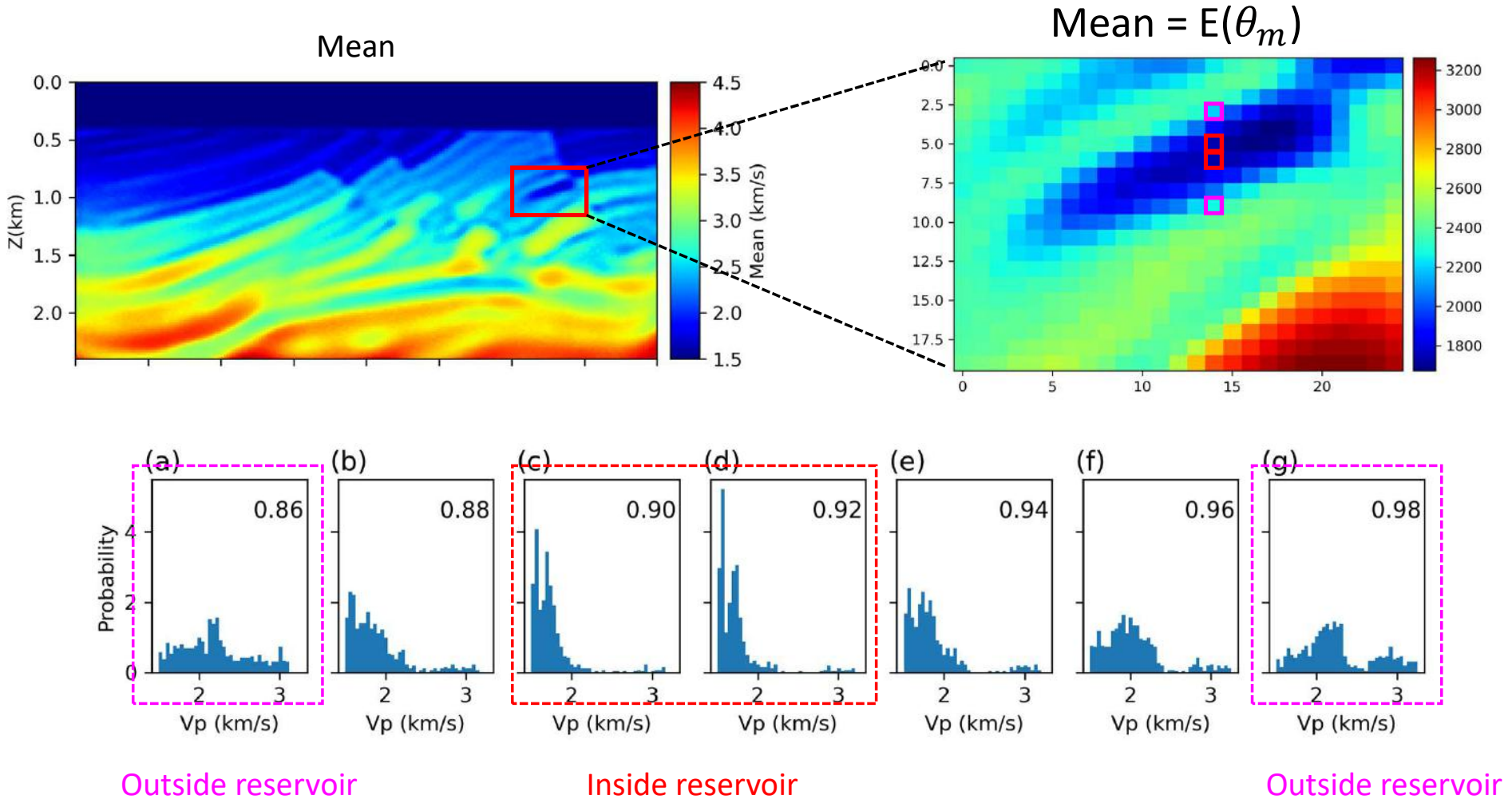
What is the size of a reservoir?



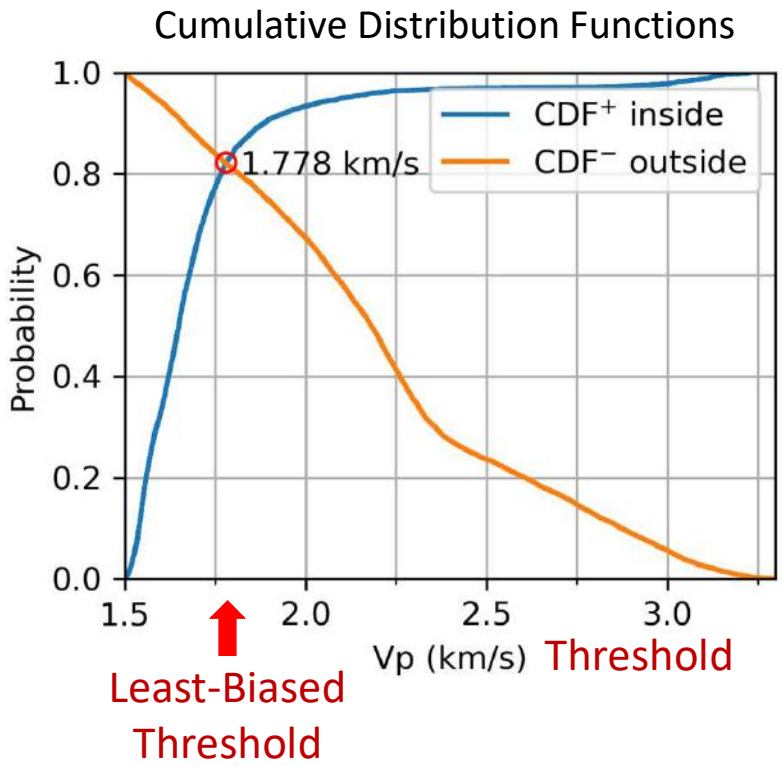
What is the size of a reservoir?

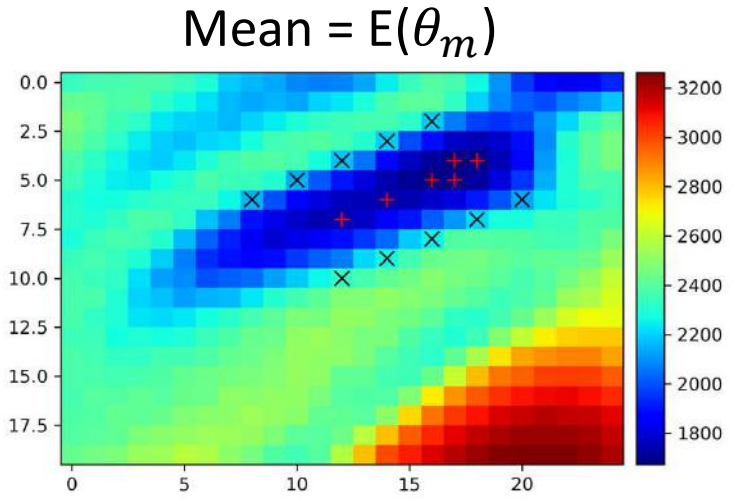


What is the size of a reservoir?



What is the size of a reservoir?



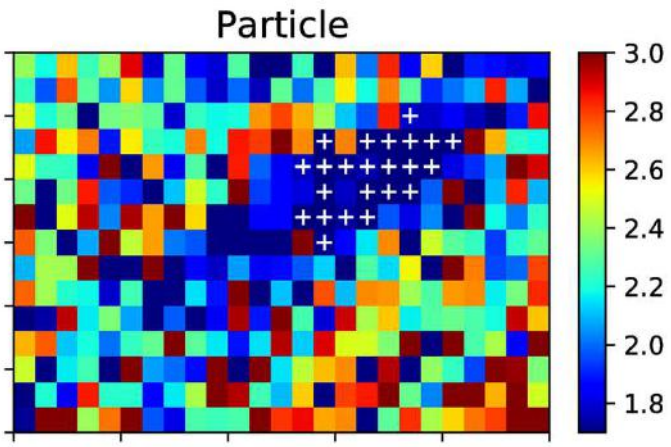


Zhang & Curtis (2022), Geophys. J. Int.
Zhao et al., (2022), J. Geophys. Res.

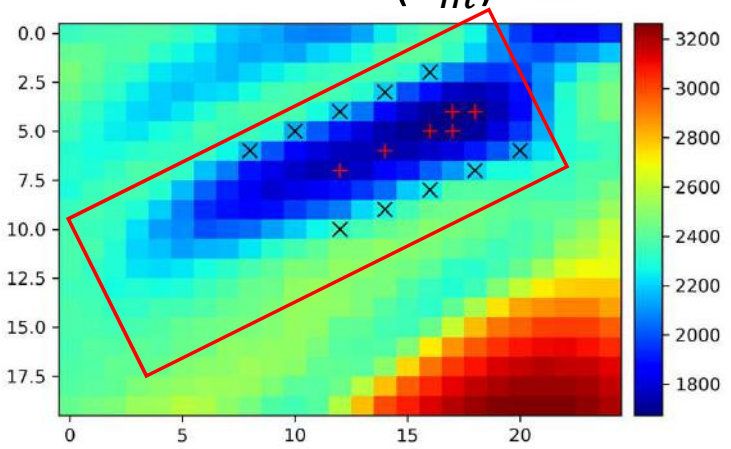
$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m) p(\theta_m, m|y_d, d)\, d\theta_m$$

T = size of the largest continuous body
in the model with velocity < 1.778 km/s

What is the size of a reservoir?



Mean = E(θ_m)

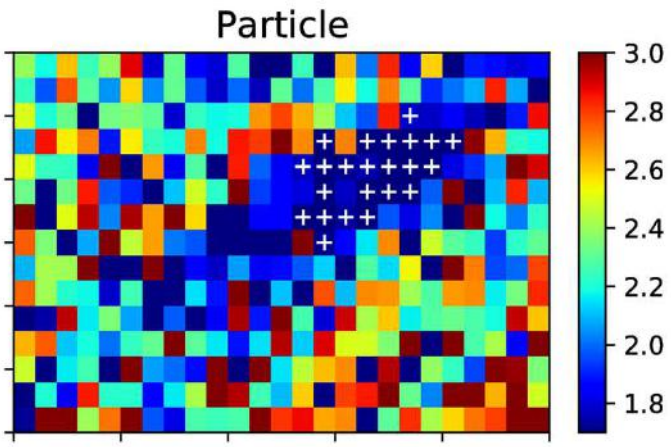


Zhang & Curtis (2022), Geophys. J. Int.
Zhao et al., (2022), J. Geophys. Res.

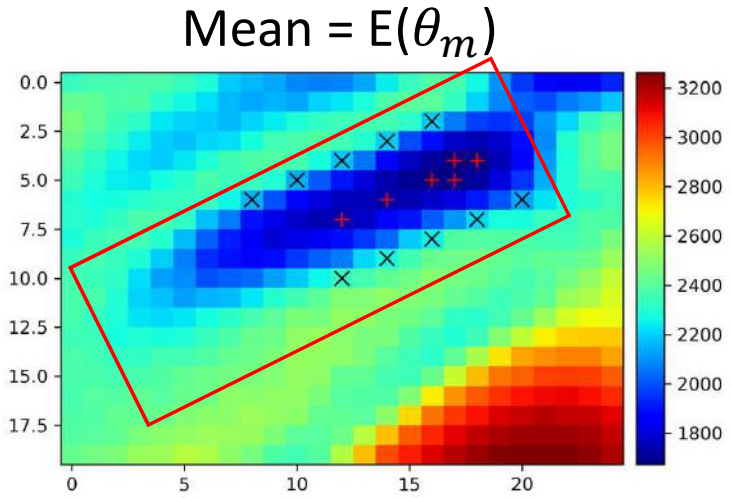
$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m) p(\theta_m, m|y_d, d)\, d\theta_m$$

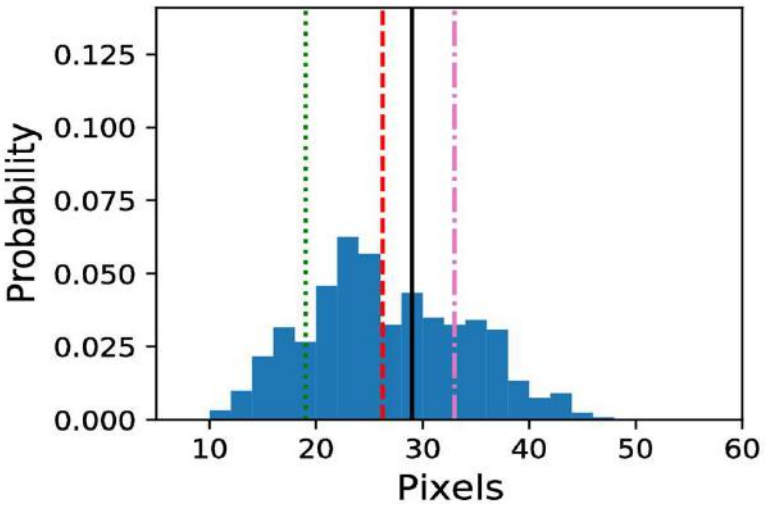
T = size of the largest continuous body with velocity < 1.778 km/s

What is the size of a reservoir?



Mean = E(θ_m)





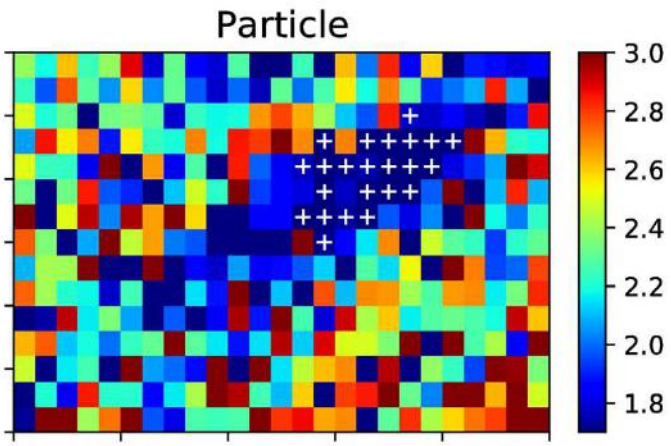
True

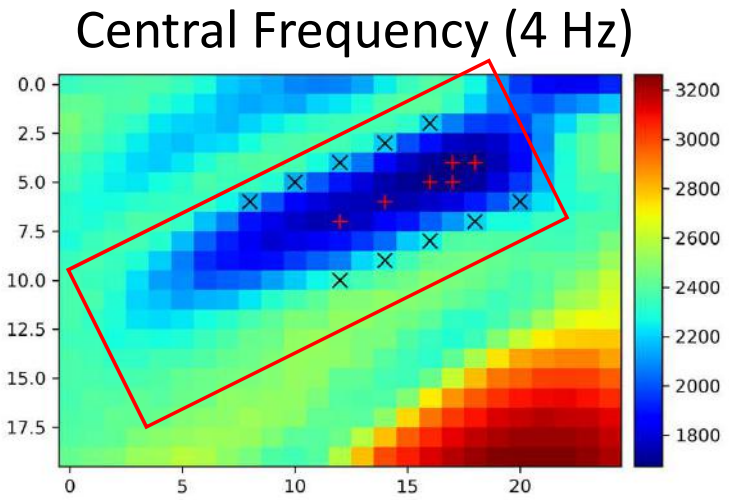
Optimal answer from Interrogation Theory

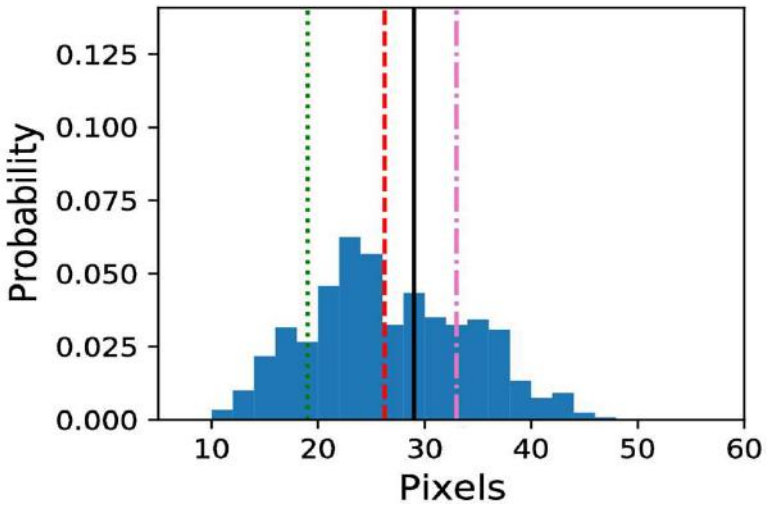
Answer obtained by interpreting mean structure

Answer obtained by interpreting median structure

What is the size of a reservoir?







True

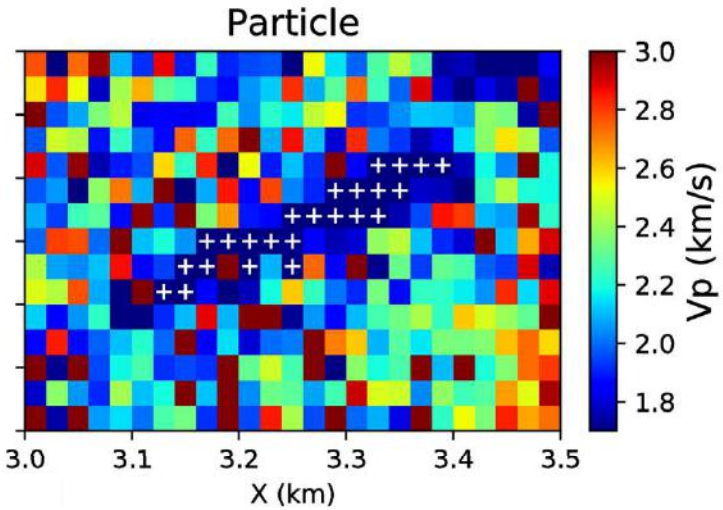
Optimal answer from Interrogation Theory

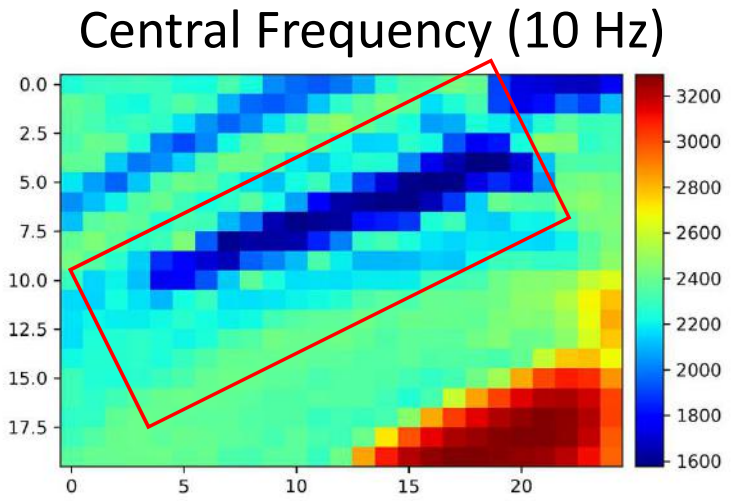
Answer obtained by interpreting mean structure

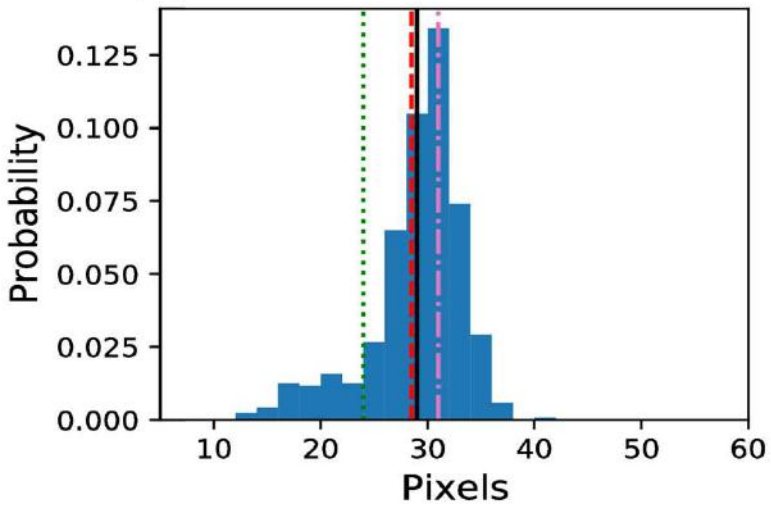
Answer obtained by interpreting median structure

Increase the frequency…

What is the size of a reservoir?







True

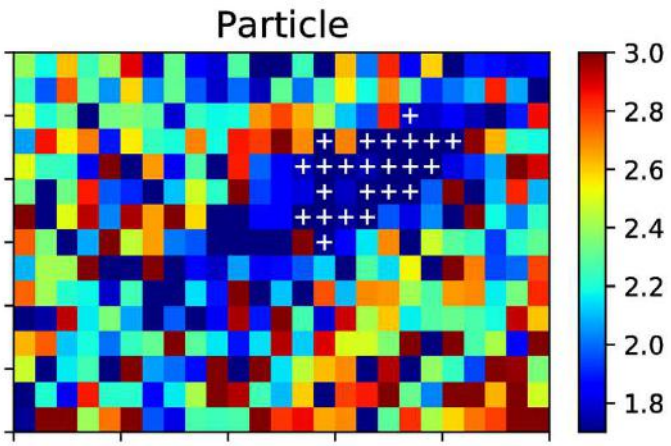
Optimal answer from Interrogation Theory

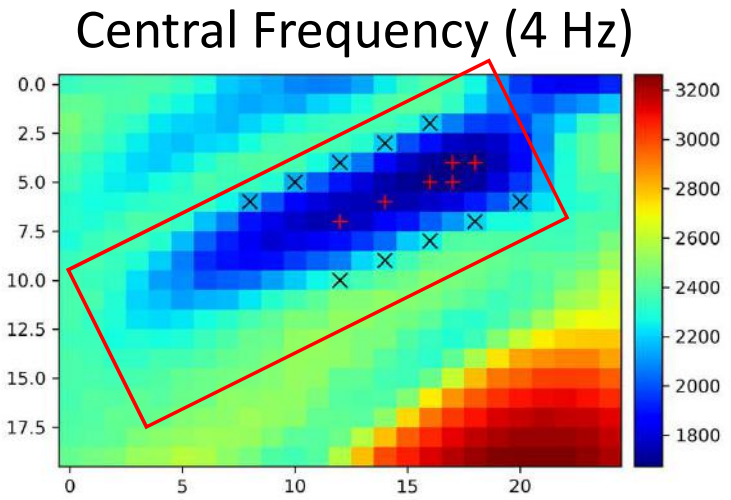
Answer obtained by interpreting mean structure

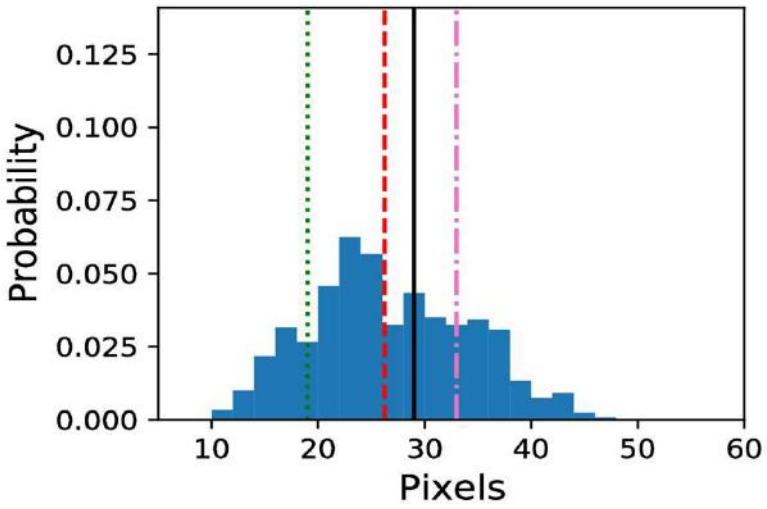
Answer obtained by interpreting median structure

Interrogation Theory provides least biased estimates from real data and allows Value of Information estimates for specific Questions

What is the size of a reservoir?







True

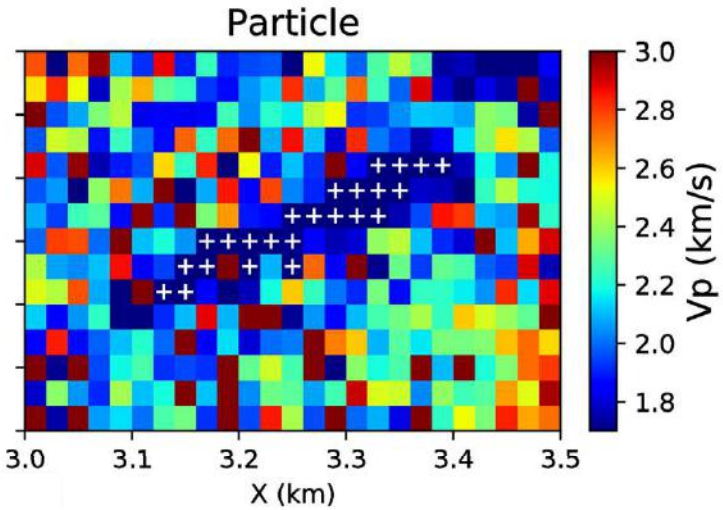
Optimal answer from Interrogation Theory

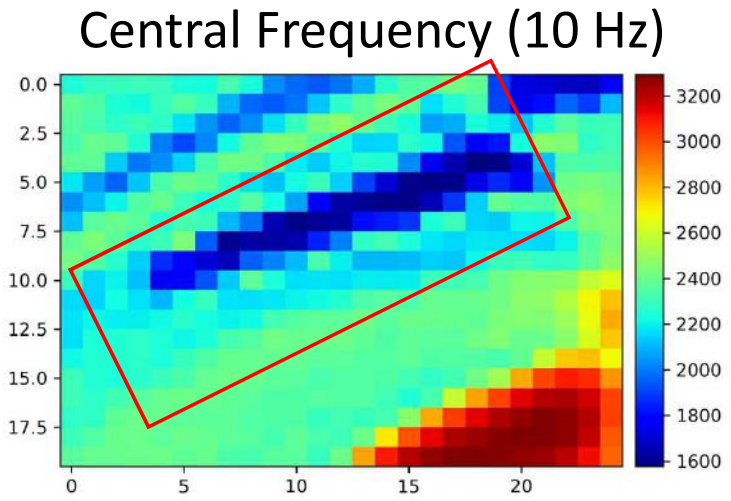
Answer obtained by interpreting mean structure

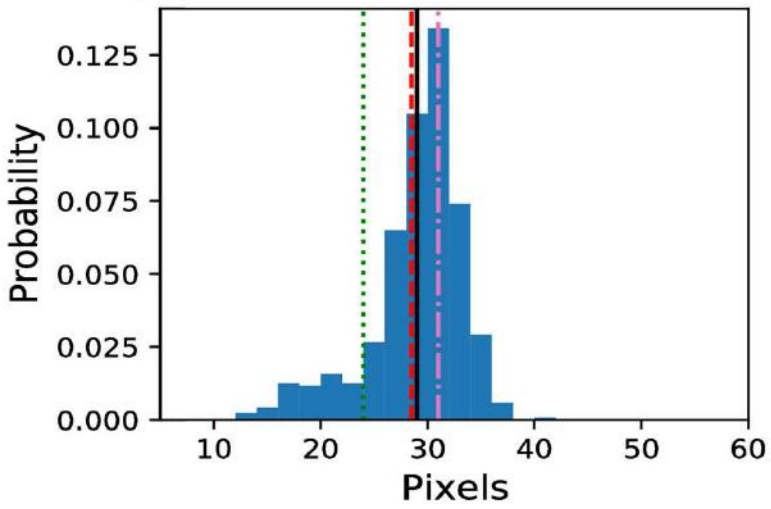
Answer obtained by interpreting median structure

Interrogation Theory provides least biased estimates from real data and allows Value of Information estimates for specific Questions

What is the size of a reservoir?







True

Optimal answer from Interrogation Theory

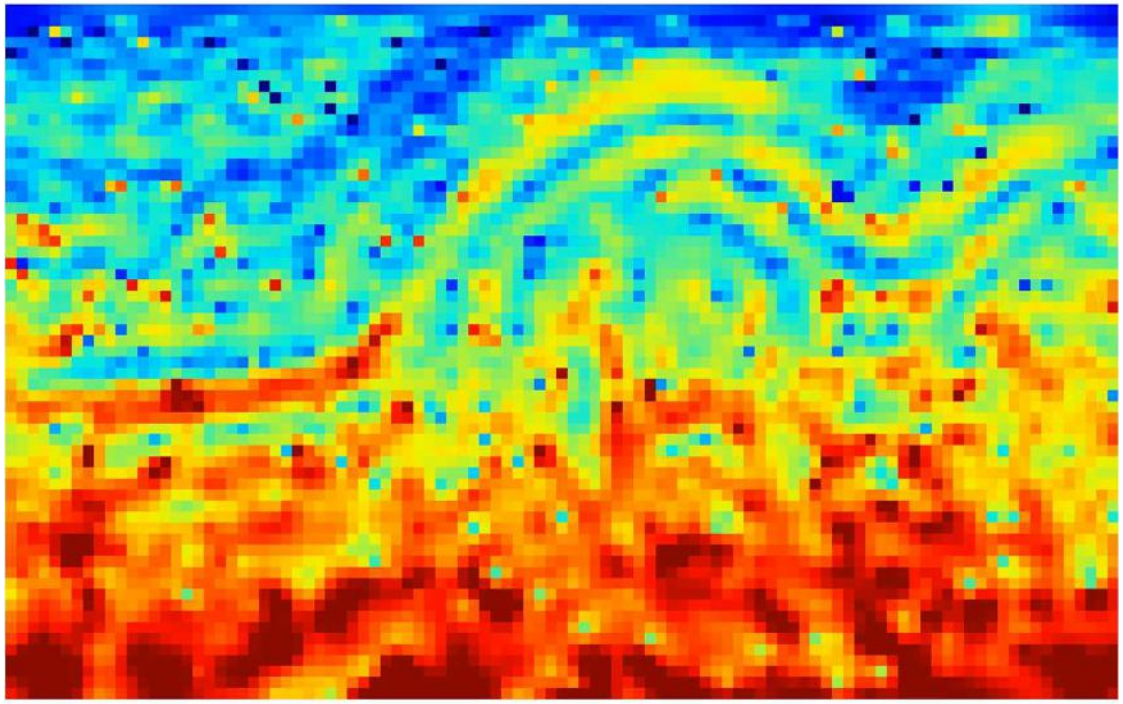
Answer obtained by interpreting mean structure

Answer obtained by interpreting median structure

Interrogation Theory provides least biased estimates from real data and allows Value of Information estimates for specific Questions

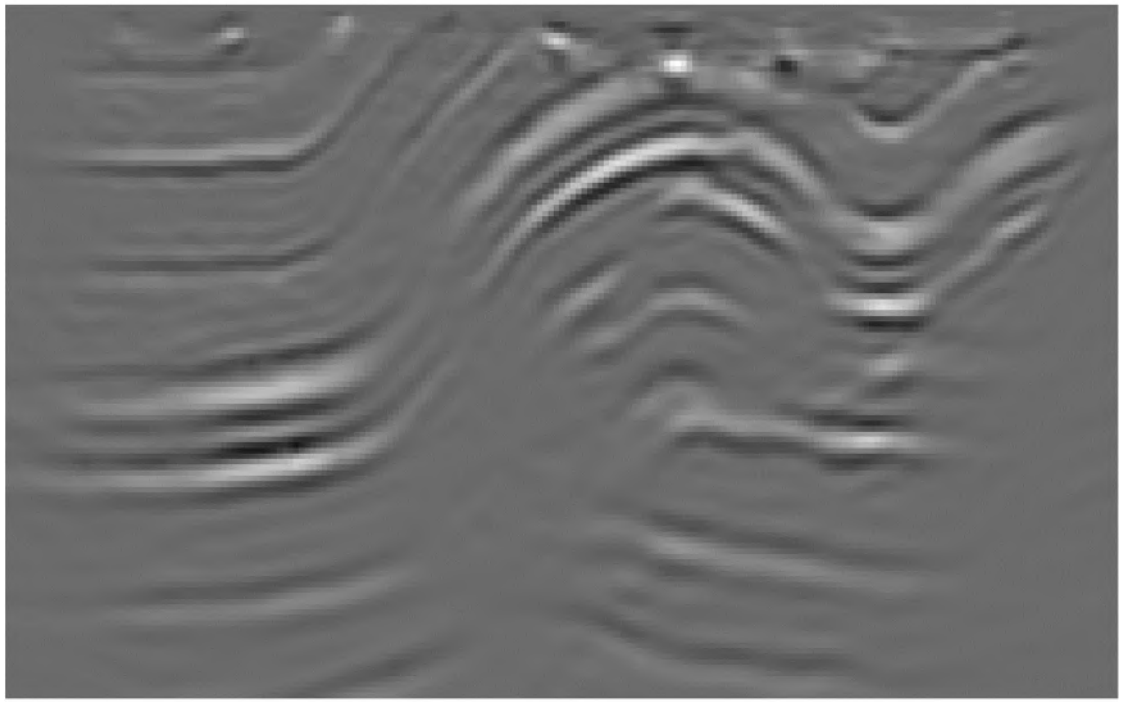
Alternative Target Functions

Movie of all particles



Alternative Target Functions

Movie of migration images of all particles



Alternative Target Functions

Interpret migration movie

$$a^* = \sum_{m \in \mathbb{M}} \int_{\theta_m} T(\theta_m|m) p(\theta_m, m|y_d, d)\, d\theta_m$$

Interim Discussion: Extensions to 3D & 4D

- Need models that represent 3D-world p(**m**|**d**)

- Previous movies were 2D slices from a 3D test model
  - Required 3D Bayesian *variational* FWI
  - Only algorithm that may work: stochastic Stein variational gradient descent (*sSVGD*)

  See: ***Zhang et al., 2022***: "3D Bayesian variational Full Waveform Inversion" (*arXiv / Geophys. J. Int., in press)*

- 3D Interrogation using FWI is computationally feasible, for small data sets

- Currently working on 4D / repeated-seismic-survey example
  - For energy transition applications, data sets will be small / sparse / focussed
  - Compared to cost of acquisition, **£-**cost of computation remains 'small' but **CO_2**-cost increasing

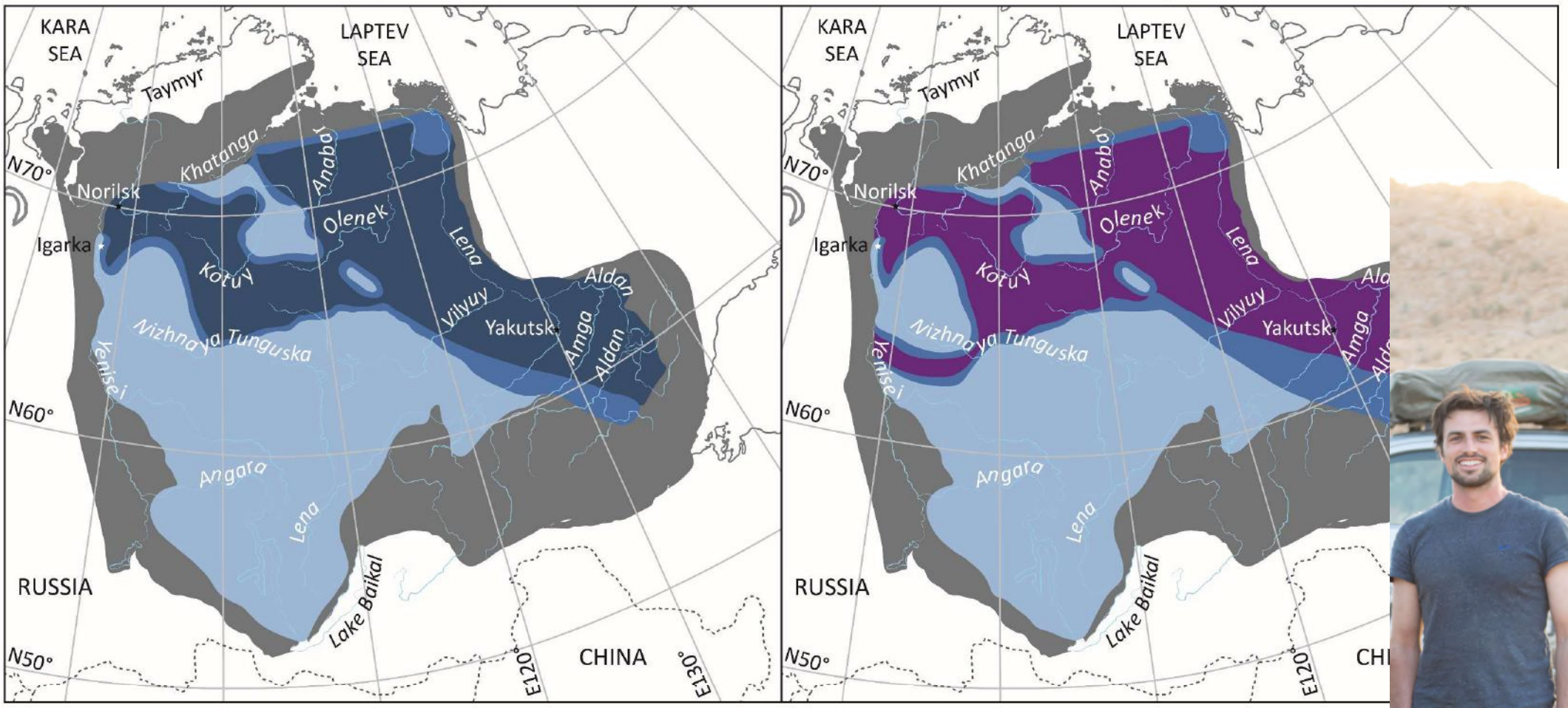
Interrogation Theory

- What is the volume of a particular subsurface reservoir? → *X* km^3
- Has more than *1 Kt* of CO_2 escaped from the subsurface store? → *Yes/No*
- Which model best explains the *true* distribution of seismicity? → *Model 1,2,3…*

In principle, all are answerable using similar quantitative models and methods

➔ **However, questions that affect the world more broadly may be different in nature…**

Narrative



Palaeofacies (Sukhov et al., 2021):
- Siberian craton
- Saliniferous facies
- Transitional facies
- Open marine carbonate facies
- Organic-rich carbonate and shale

Siberia in the Cambrian:

Two theories explain the distribution of oxygen: **Theory A & Theory B**

<u>Everyone</u> please choose which theory they think is most likely to be true.

Theory A or **Theory B**

Which is most likely to be true?

Theory A or **Theory B**

Which is more **emotionally** satisfying, **fulfilling** or appealing, or gives more **positive feelings** of any kind?
Which one do you **like**?

Narrative

Raise a hand if the **same hand** was <u>both</u>
more likely to be true,
and most emotionally satisfying

i.e., if you squeezed the same hand twice

Interrogation Theory

- What is the volume of a particular subsurface reservoir? → *X* km^3
- Has more than *1 Kt* of CO_2 escaped from the subsurface store? → *Yes/No*
- Which model best explains the *true* distribution of seismicity? → *Model 1,2,3…*

Interrogation Theory

- What is the volume of a particular subsurface reservoir? → *X km^3*
- **Should we buy a license to store CO_2?** → *Yes/No*
- **Should a business enter a new sector: storing Hydrogen or CO_2?** → *Yes/No*

The questions lie outside of 'model world' – they lie inextricably in the real world

- Will there **be** a Hydrogen storage market?
- What will future interest, tax and discount rates be over next 10 years?
- How rapidly can/will society's reliance on hydrocarbons abate?
- Will the most experienced staff, key investors, or voters be inspired, or concerned?
- **How will my decision change the world itself?**

→ Escape from Model World (Erica Thompson, 2022)

- What is the volume of a particular subsurface reservoir? → *X km^3*
- **Should we buy a license to store CO_2?** → *Yes/No*
- **Should a business enter a new sector: storing Hydrogen or CO_2?** → *Yes/No*

The questions lie outside of 'model world' – they lie inextricably in the real world

- Will there **be** a Hydrogen storage market?
- What will future interest, tax and discount rates be over next 10 years?
- How rapidly can/will society's reliance on hydrocarbons abate?
- Will the most experienced staff, key investors, or voters be inspired, or concerned?
- **How will my decision change the world itself?**

Bayes' Theorem

Model World

Real World

Credit Card
Interest Rates
28% APR

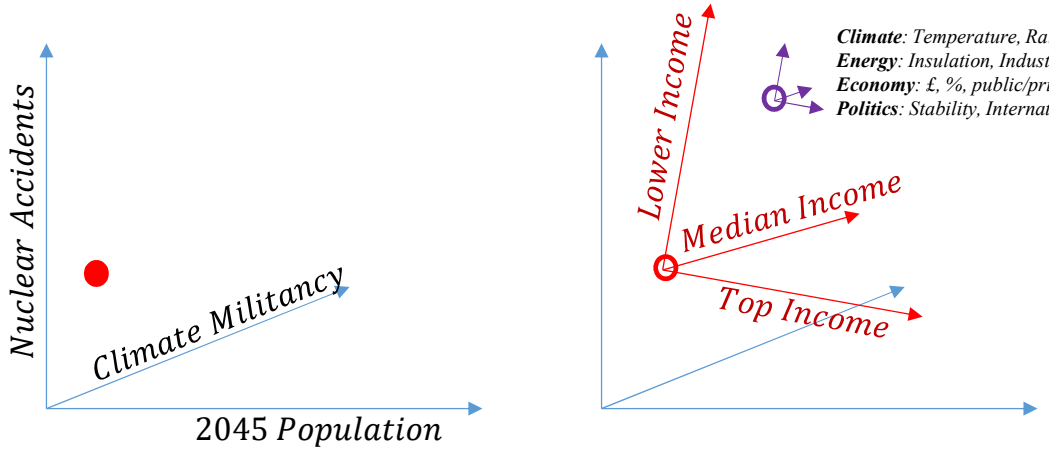
Model World

Real World

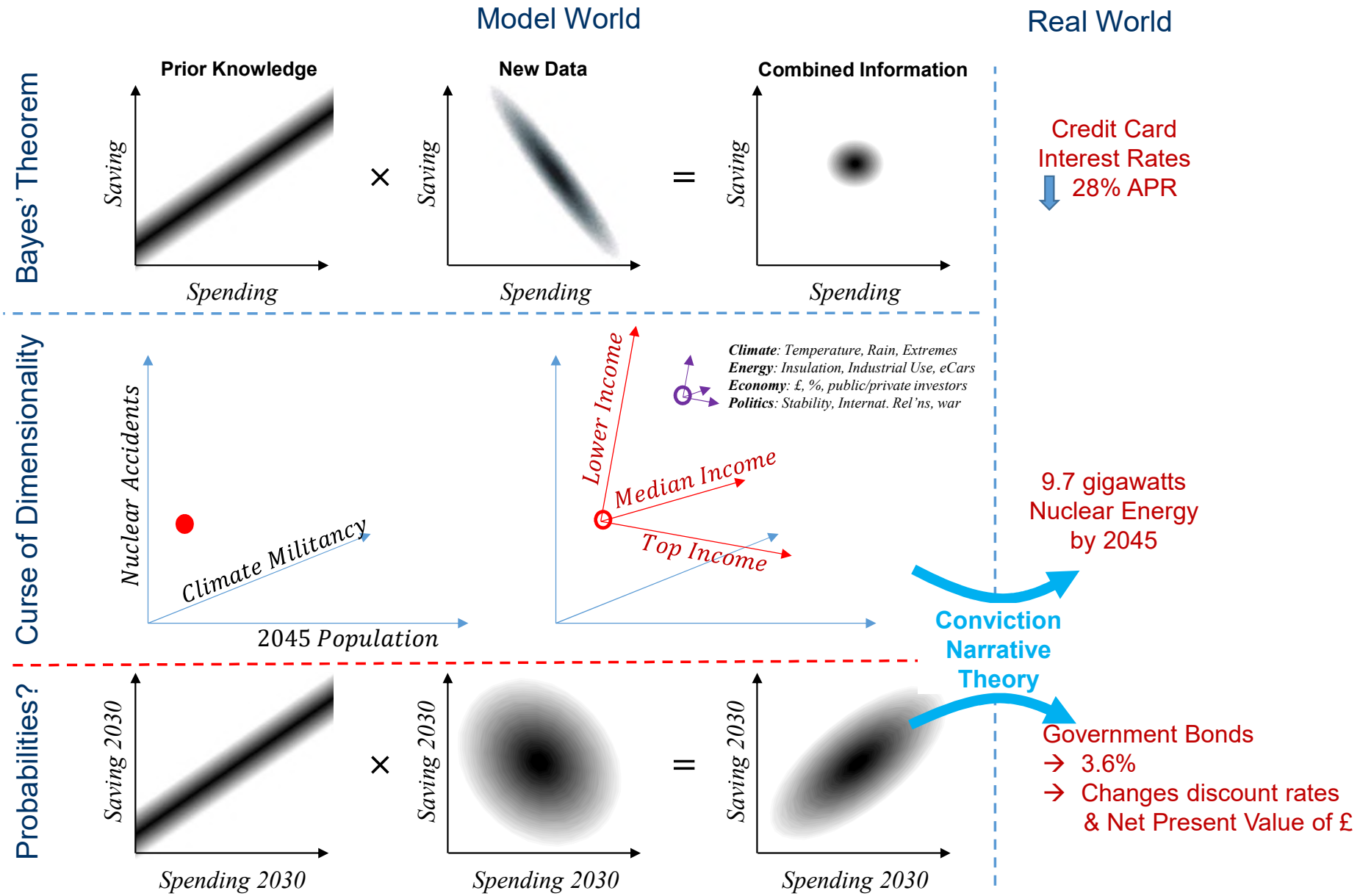
Bayes' Theorem

Credit Card Interest Rates ⬇ 28% APR

Curse of Dimensionality



9.7 gigawatts Nuclear Energy by 2045


Model World
Real World
Bayes' Theorem
Prior Knowledge
New Data
Combined Information
Saving
Spending
×
=
Credit Card
Interest Rates
28% APR
Curse of Dimensionality
Nuclear Accidents
Climate Militancy
2045 Population
Lower Income
Median Income
Top Income
Climate: Temperature, Rain, Extremes
Energy: Insulation, Industrial Use, eCars
Economy: £, %, public/private investors
Politics: Stability, Internat. Rel'ns, war
9.7 gigawatts
Nuclear Energy
by 2045
Conviction
Narrative
Theory
Probabilities?
Saving 2030
Spending 2030
Government Bonds
→ 3.6%
→ Changes discount rates
& Net Present Value of £

Conviction Narrative Theory

Johnson et al., (2022)
Psychologists, Decision Theorists, Management Scientists

Humans don't explore and make optimal decisions

- Under radical uncertainty, people tend to explore few options (even only 1). **They make decisions based on *narratives*.**
- **Narratives**: mental representations that summarise relevant causal, temporal, analogical or valence information. (In short: *stories*)

A narrative is selected which *explains past/present* data, is then used to *imagine possible futures*; **emotional** reactions to the narrative provide *affective evaluation*.
→ Choose narrative that produces *conviction to take sustained action*.

"Emotions lead to biases" – Kahnemann & Tversky + many – "rational thought is evidence based"
➔ **"Emotional responses encode long-term stores of rationality"**

Conviction Narrative Theory

Johnson et al., (2022)
Psychologists, Decision Theorists, Management Scientists

Matters arising:

- Conviction Narrative Theory describes what decision-makers ***do***, not necessarily what they ***should*** do.
- This is a very different way to view '*rationality*' in decision making.
- What people do as a result may be rational in some cases – but how often?
- *What else could they do?* *(How did we do?)*
- If that is how decision-makers make decisions, **what should *we* do?**
- There is a need to develop new theory and methods that **span both worlds**

Thank you!

All of our papers are available at: https://blogs.ed.ac.uk/curtis/publications

Arnold & Curtis (2018):	"*Interrogation Theory*", Geophys. J. Int.
Zhang & Curtis (2022):	*"Interrogating probabilistic inversion results for subsurface structural information"*, Geophys. J. Int.
Zhao et al., (2022):	*"Interrogating Subsurface Structures using Probabilistic Tomography: an example assessing the volume of Irish Sea basins"*, J. Geophys. Res.
Zhang et al., (2022):	*"3D Bayesian Variational Full Waveform Inversion"*, arXiv
Johnson et al. (2022):	"Cognitive Narrative Theory: A Theory of Choice under Radical Uncertainty", Behavioural and Brain Sciences, *in press*
Thompson (2022):	*"Escape from Model Land", Basic Books, London*