Born to diffuse:
Assessing transitional scattering regimes by waveform complexity

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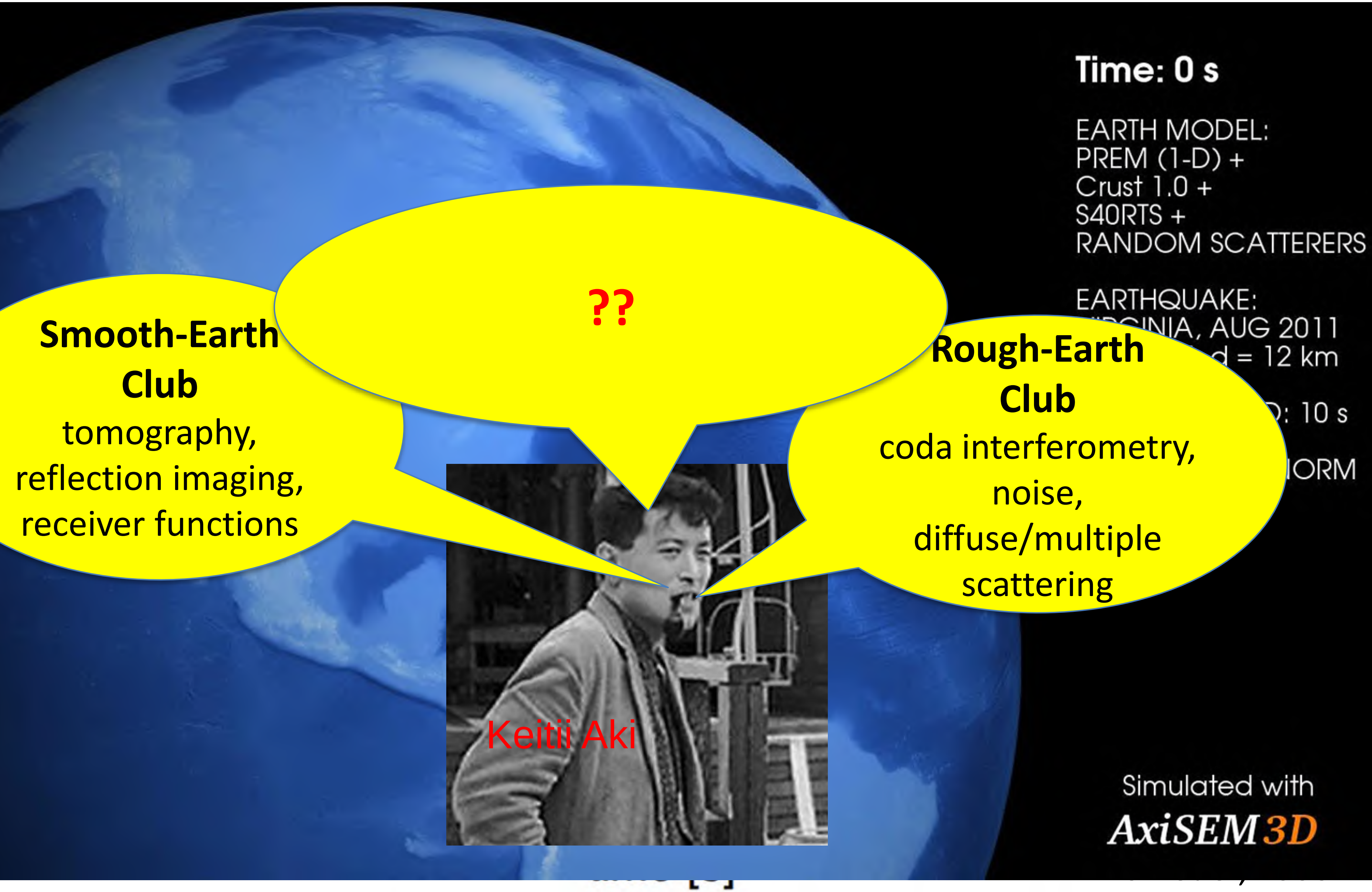
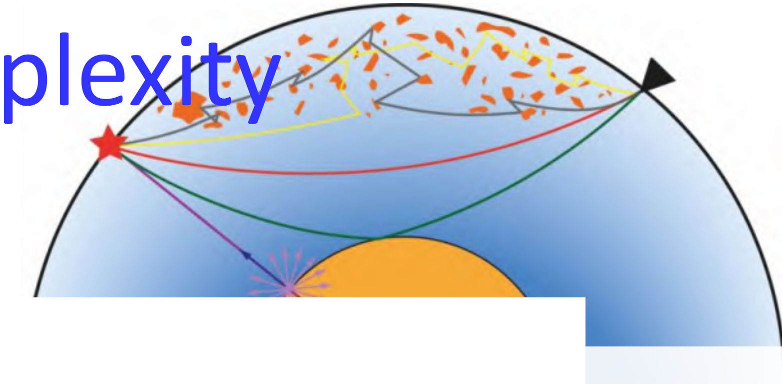
Multiscale Earths



Structure-waveform complexity



- **How do we measure waveform complexity/diffusivity?**
- **What parameters control & show the transition from ballistic to diffuse scattering?**
- **Is there a distinct transitional regime in realistic settings?**
- **Can we quantify heterogeneity characteristics along these regimes?**
- **Why is this interesting (for others than smooth/rough wave fanatics) ??**

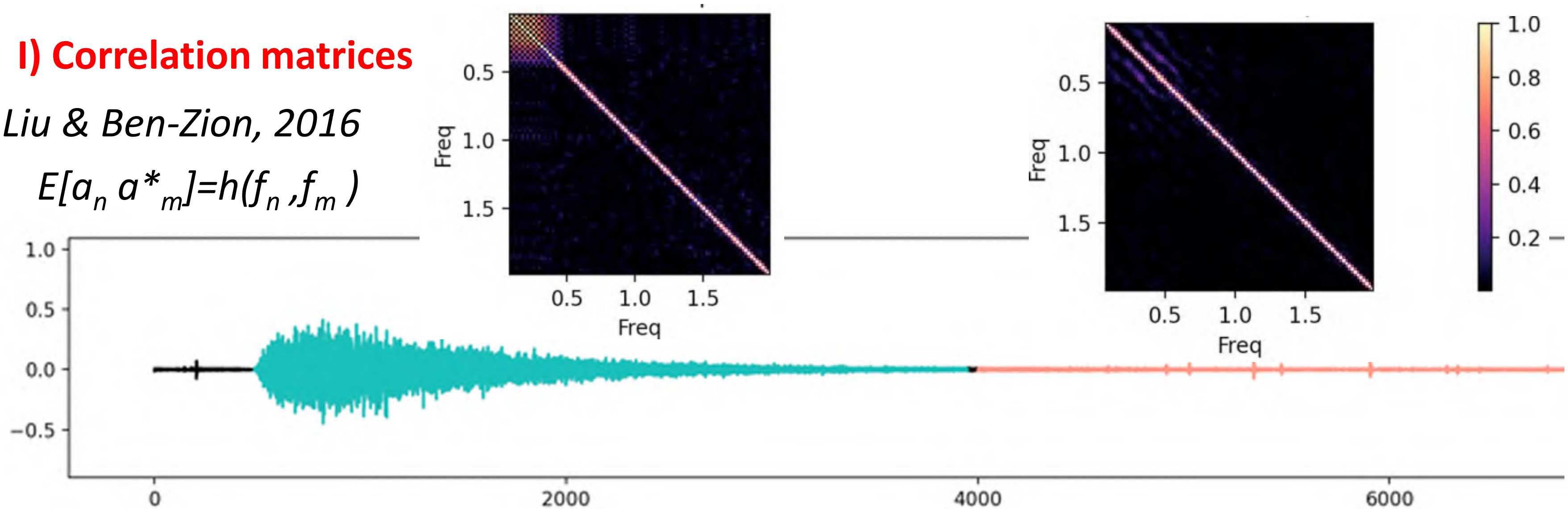
How to measure waveform complexity?

1) There is **no unique definition of complexity**, or a direct mapping to scattering objects
2) Each technique **measures a different aspect** of complexity
3) Each technique may have **different validity regimes**, depending on level of complexity

I) Correlation matrices

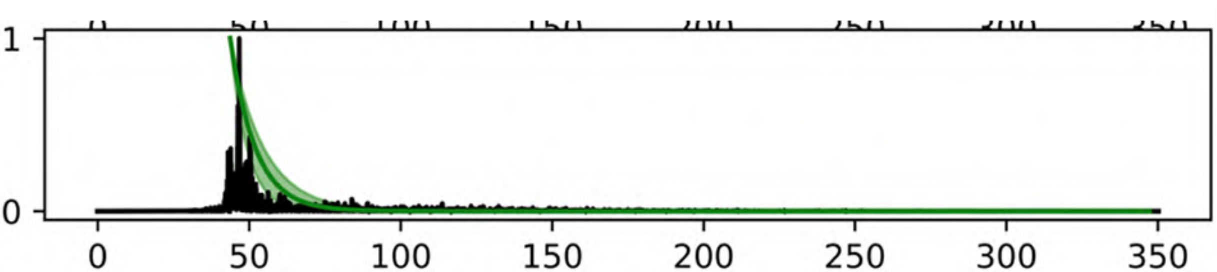
Liu & Ben-Zion, 2016

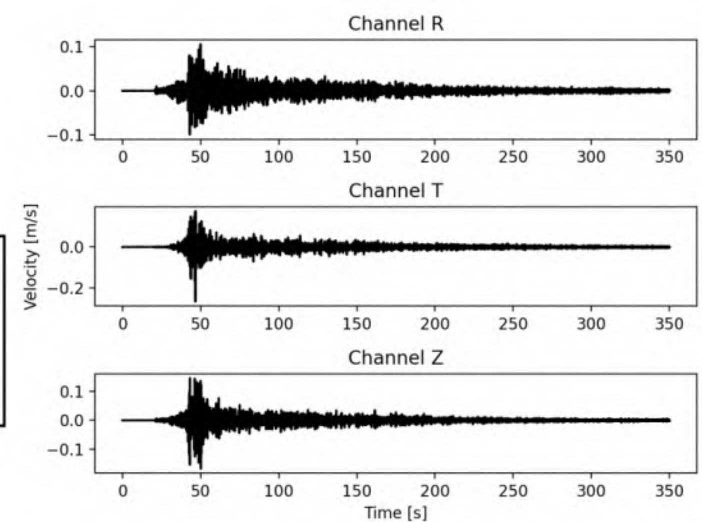
$E[a_n a^*_m]=h(f_n, f_m)$



II) Coda Q

$$\langle E(t)\rangle_T^{\text{S Coda}} \propto \frac{1}{t^n}\exp(-2Q_c^{-1}\pi f t)$$





III) Multiscale Entropy

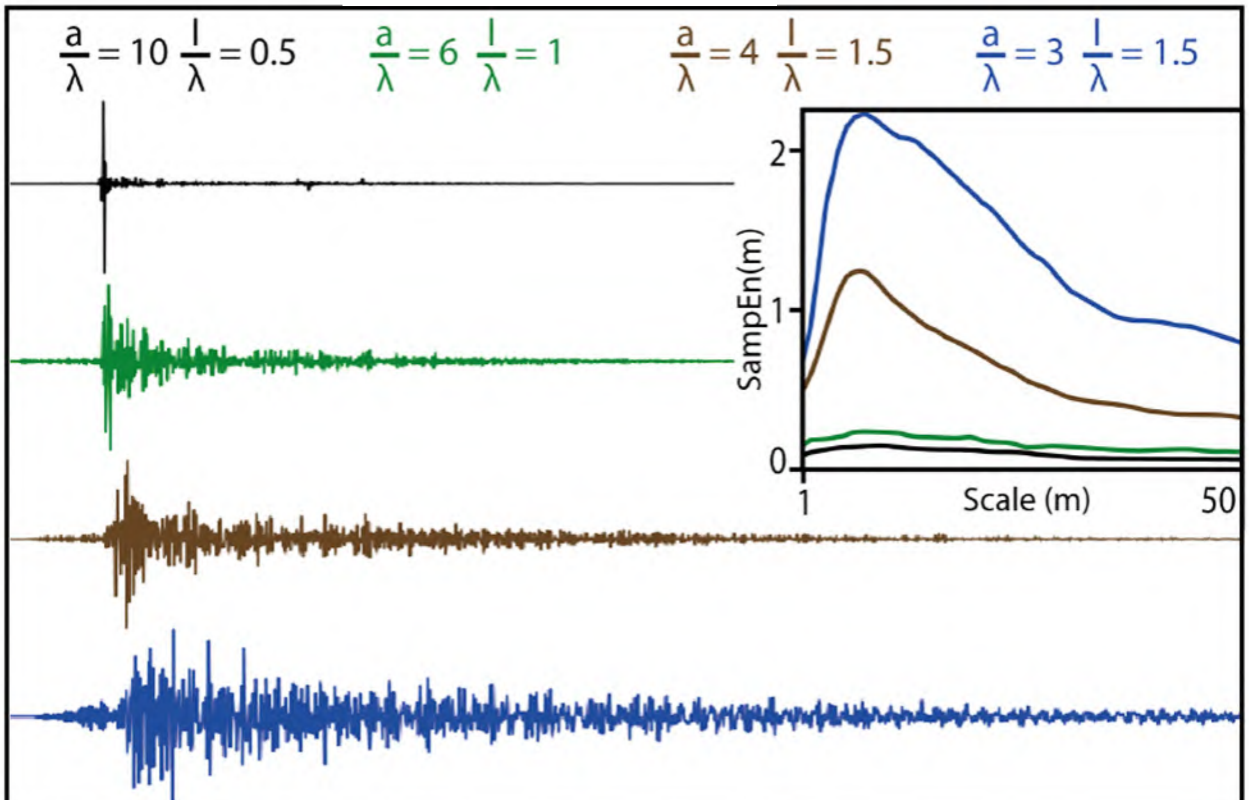
Measure difference between template vectors (length m)

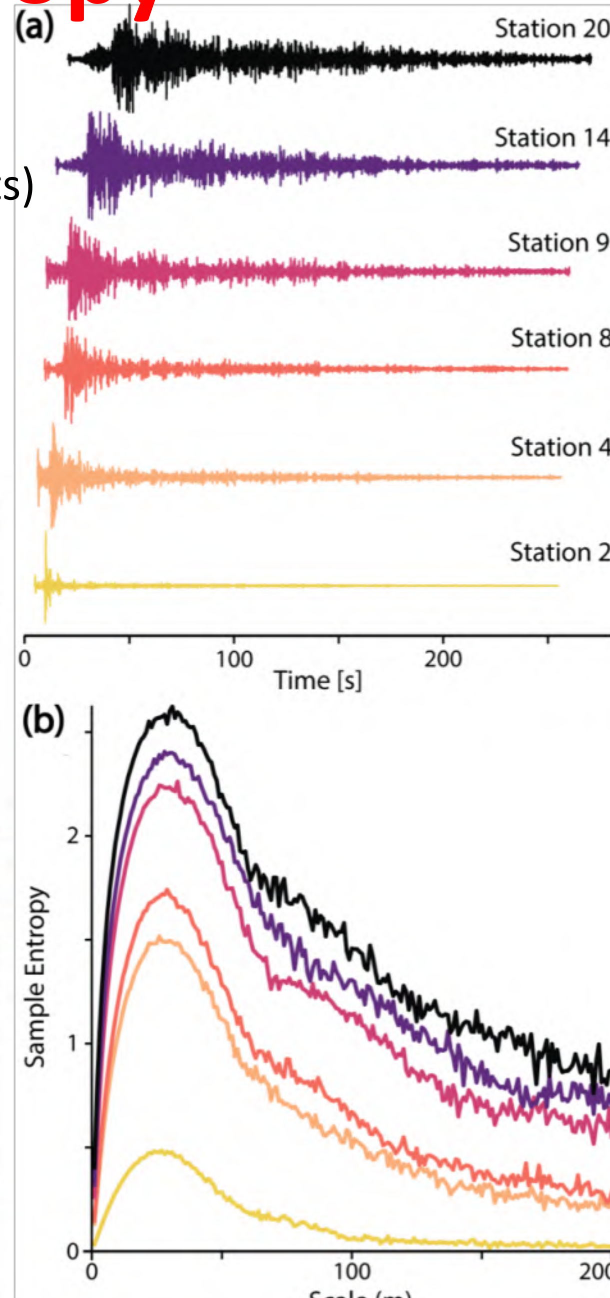
Entropy ~ log (cond. Prob. that 2 sequences matched for m points)

$$\mathbf{x}_i^m = \{u(i+h)\} \qquad \text{for} \quad 0 \le h \le (m-1)$$

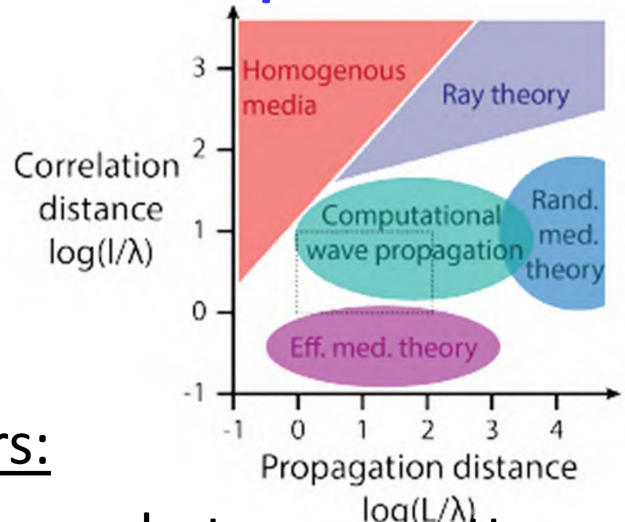
Moving time windows -> time-dependent entropy

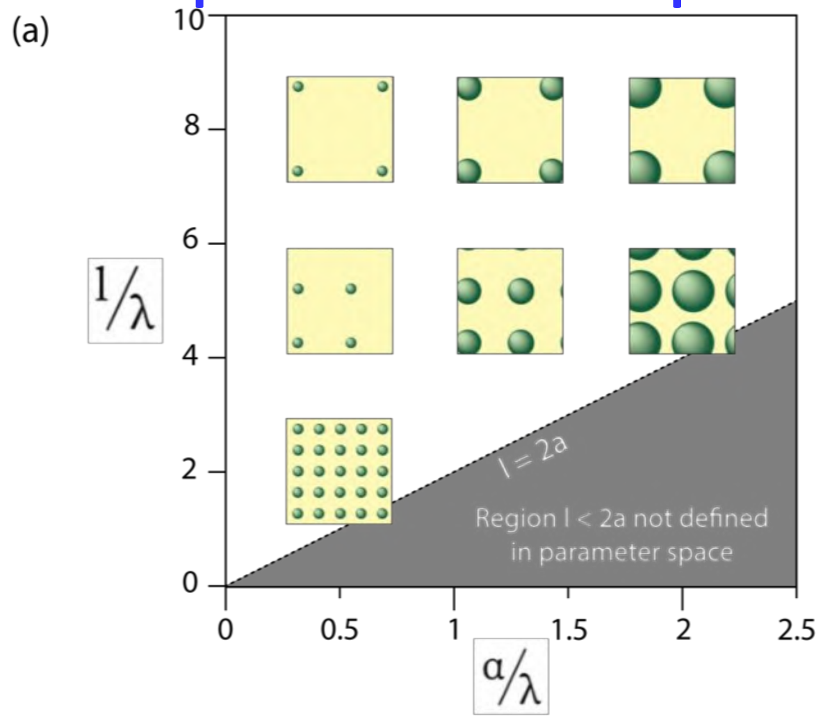
$$\mathrm{SampEn}(N, m, \sigma) \approx -\ln \frac{A^m(\sigma)}{B^m(\sigma)}$$

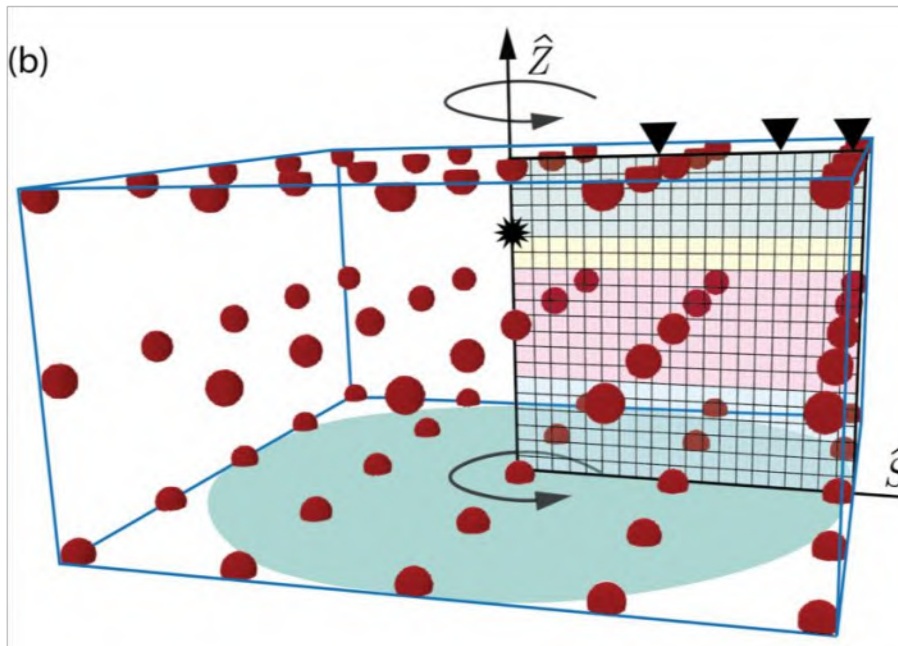




Sampling scattering regimes: a parameter-space simulation experiment







Parameters:

l – distance between scatterers

- Min: 0; Max: 10

a – radius of scatterer

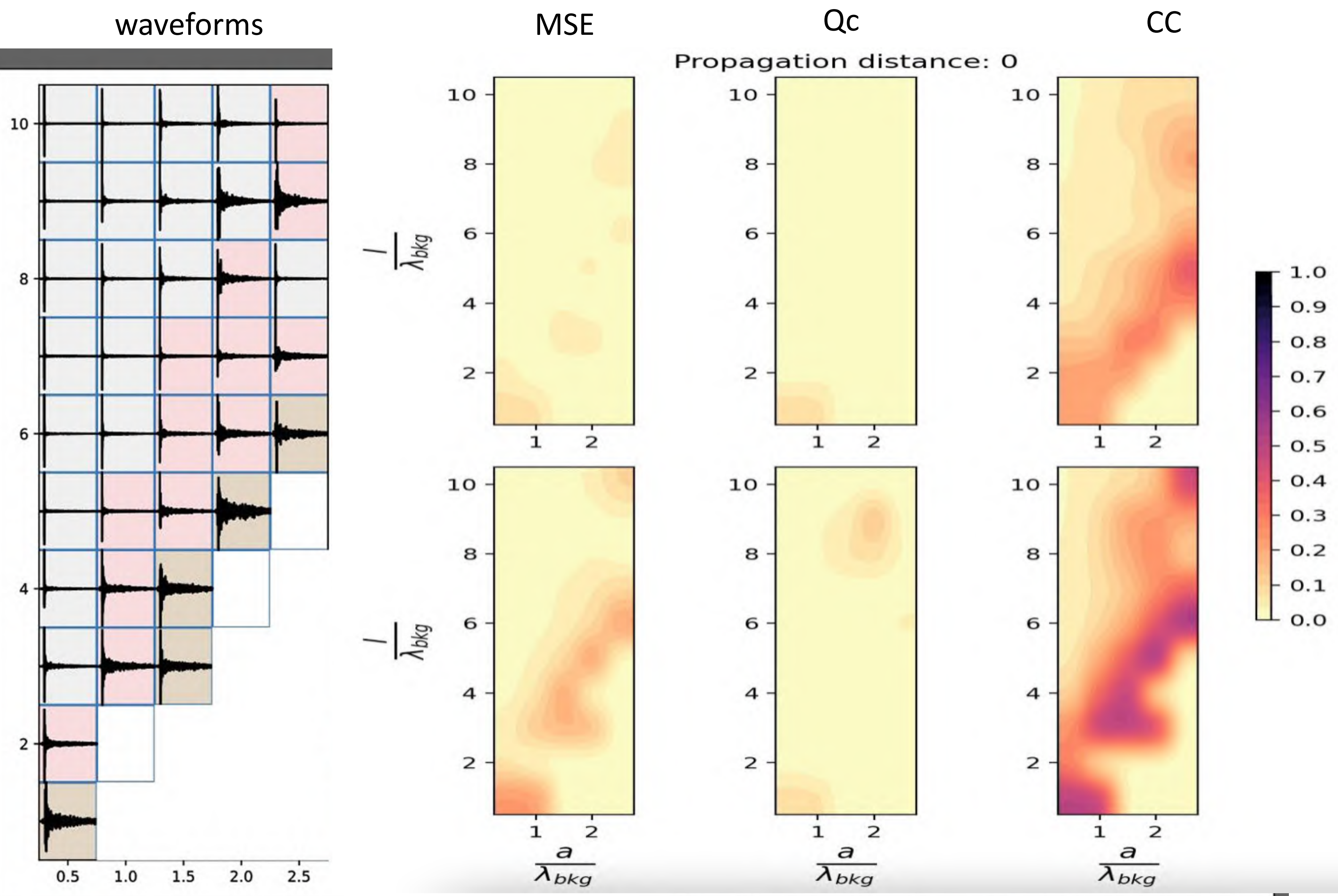
- Min: 0; Max: 2.5

δv – perturbation strength

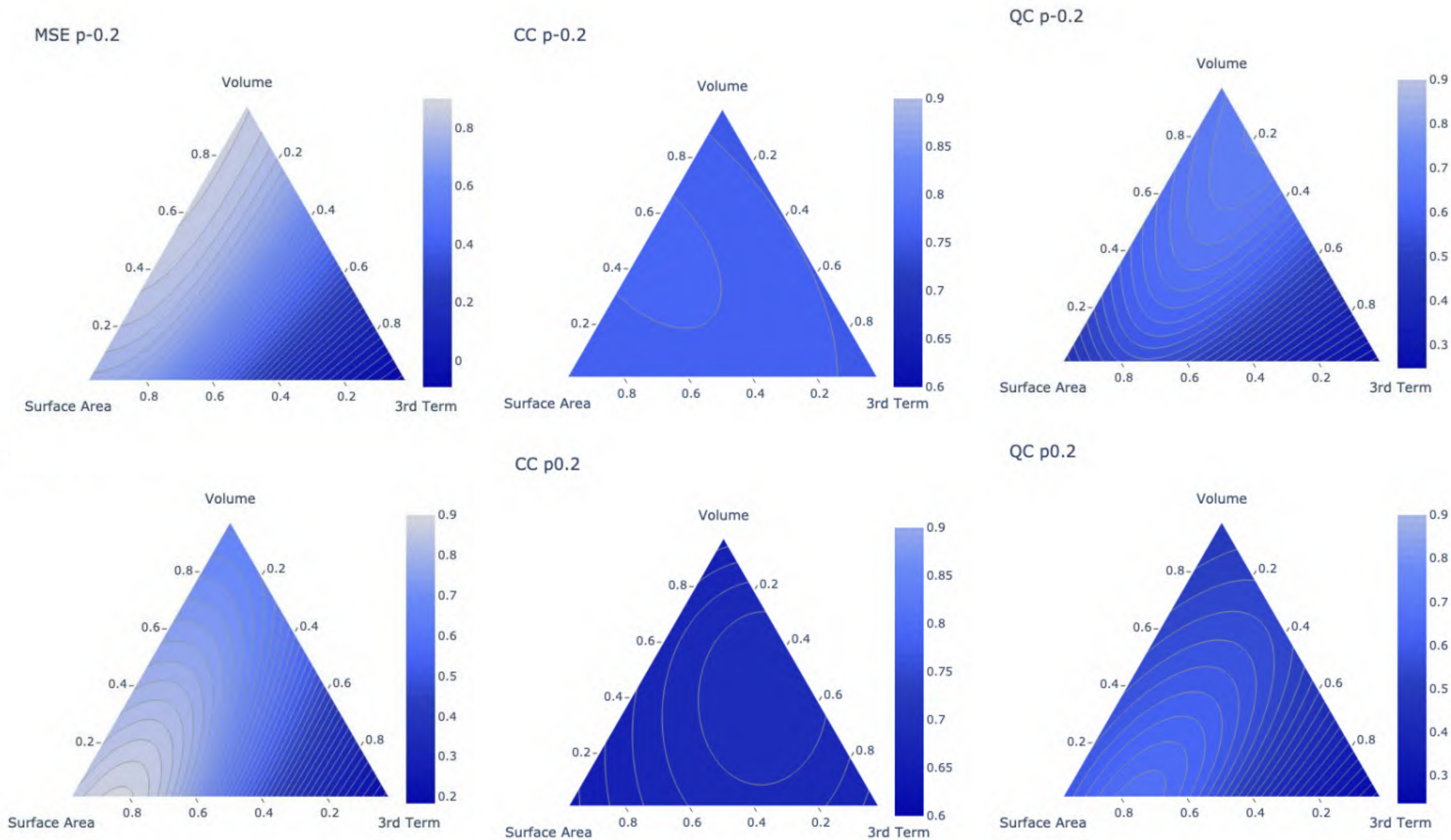
- Min: - 30 %; Max: + 40 %

L – number of wavelengths propagated

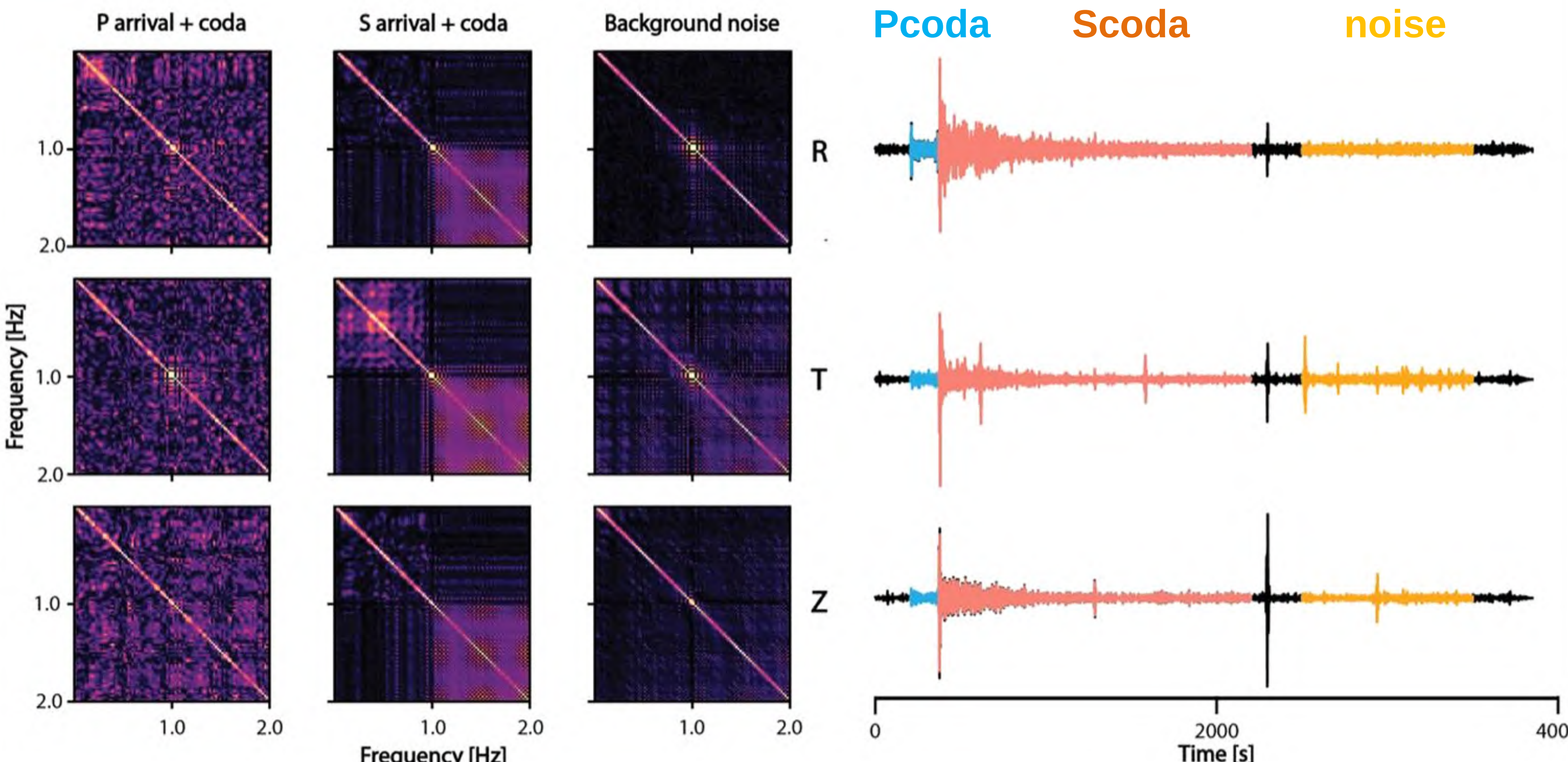
Characterising complexity regimes

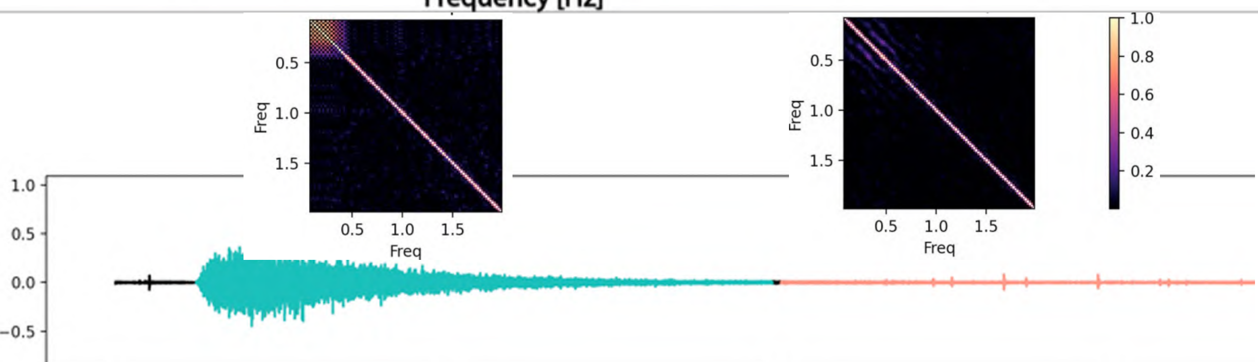


Inferring structural parameters (statistically)?

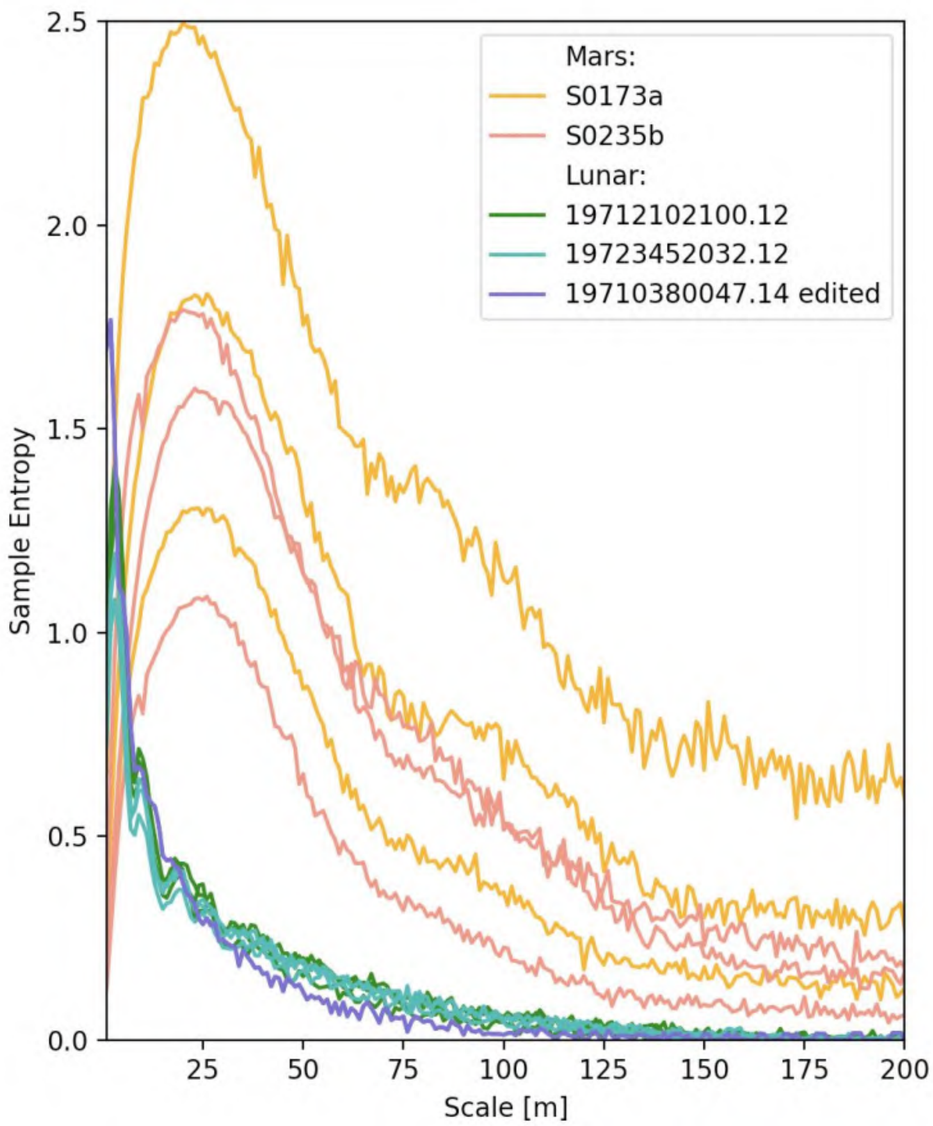


Correlation-coefficient matrices: Mars & Moon



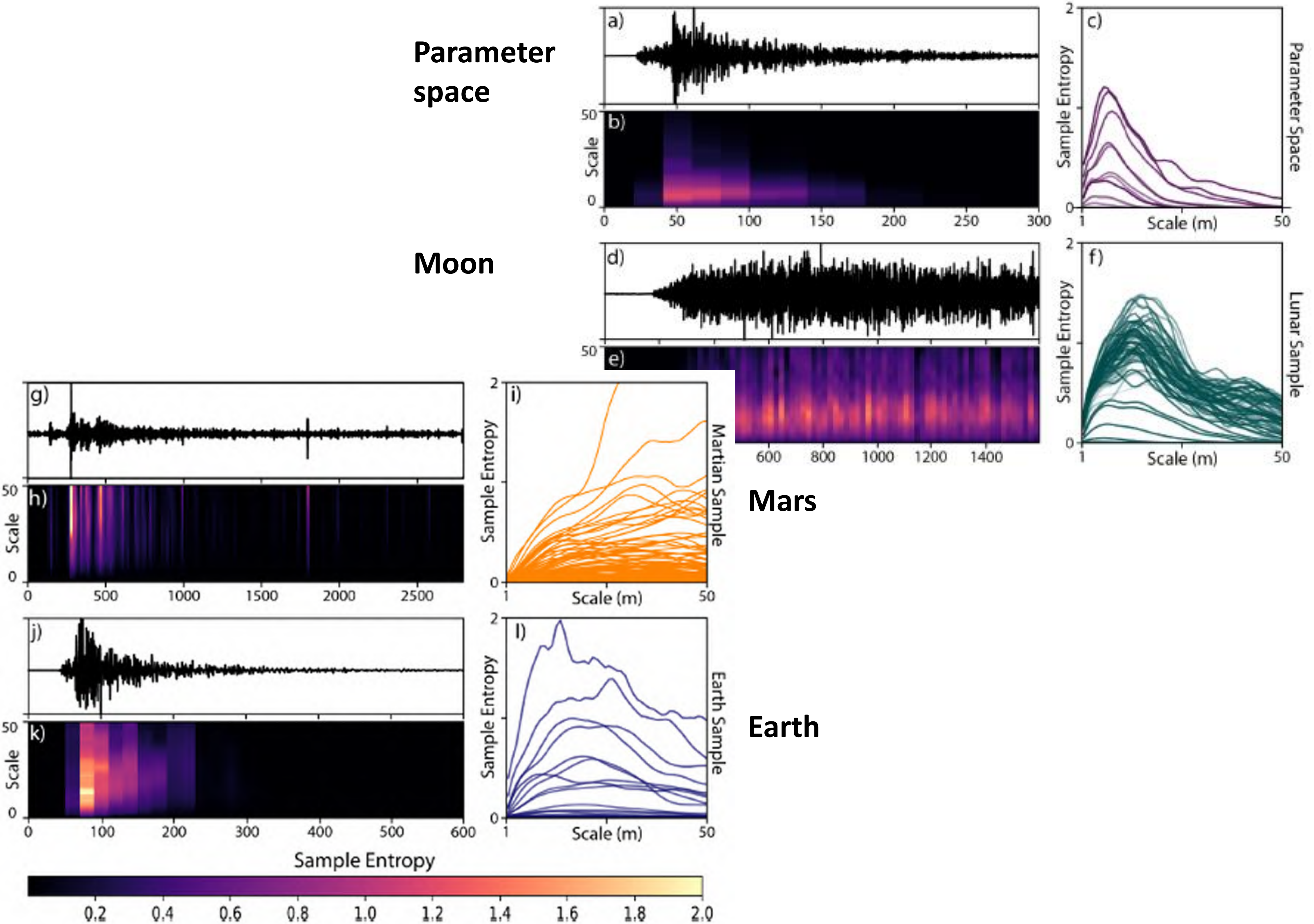


Entropy on Mars and Moon

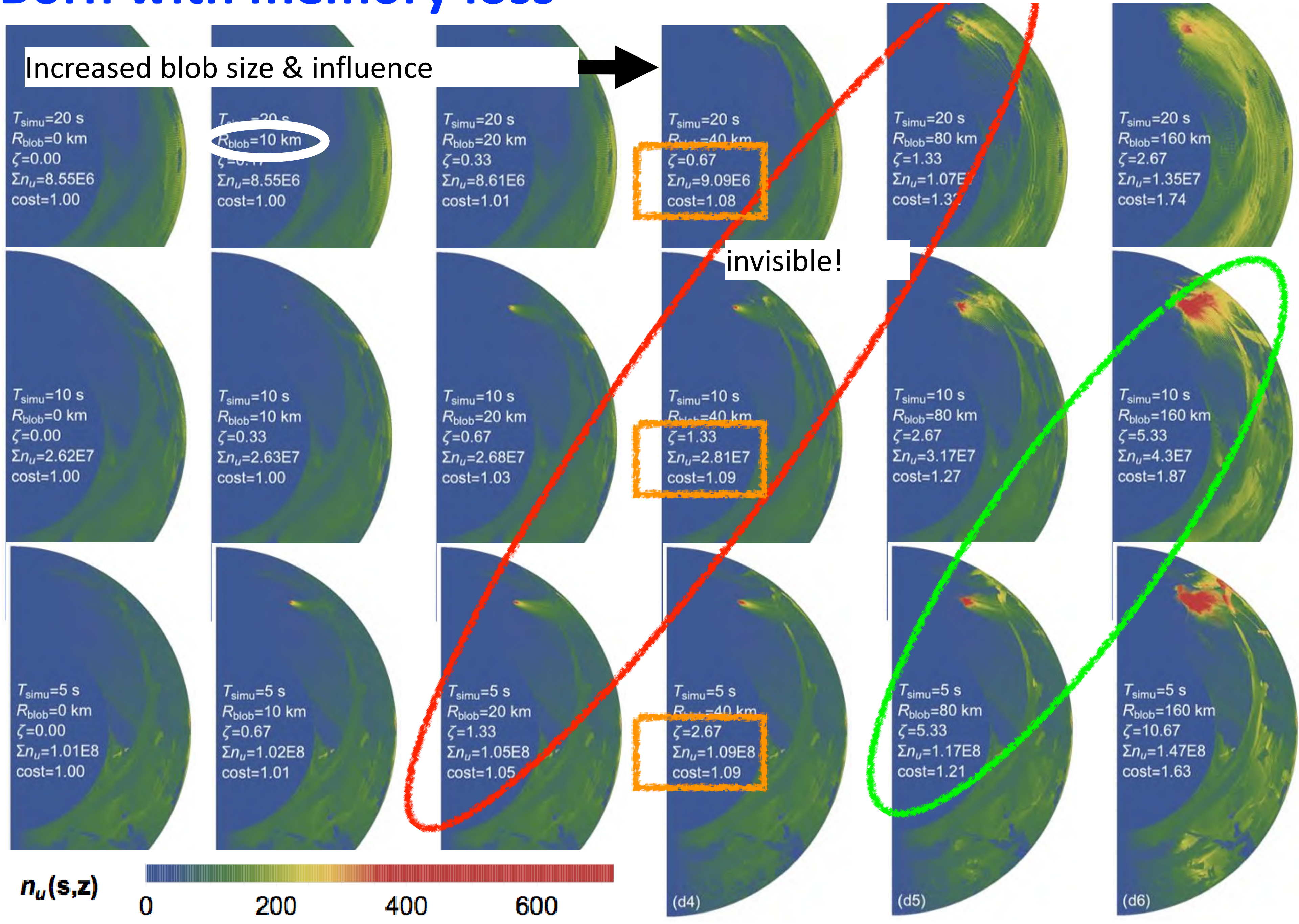


- Moon entropy increases w bandpass
- Mars MSE higher: incoherent scattering?
- Variation between events, although similar x
- Mars entropy peak at 1Hz
- more varied between channels: anisotropy?
- Issues: glitches, sampling rates, Moon xyz

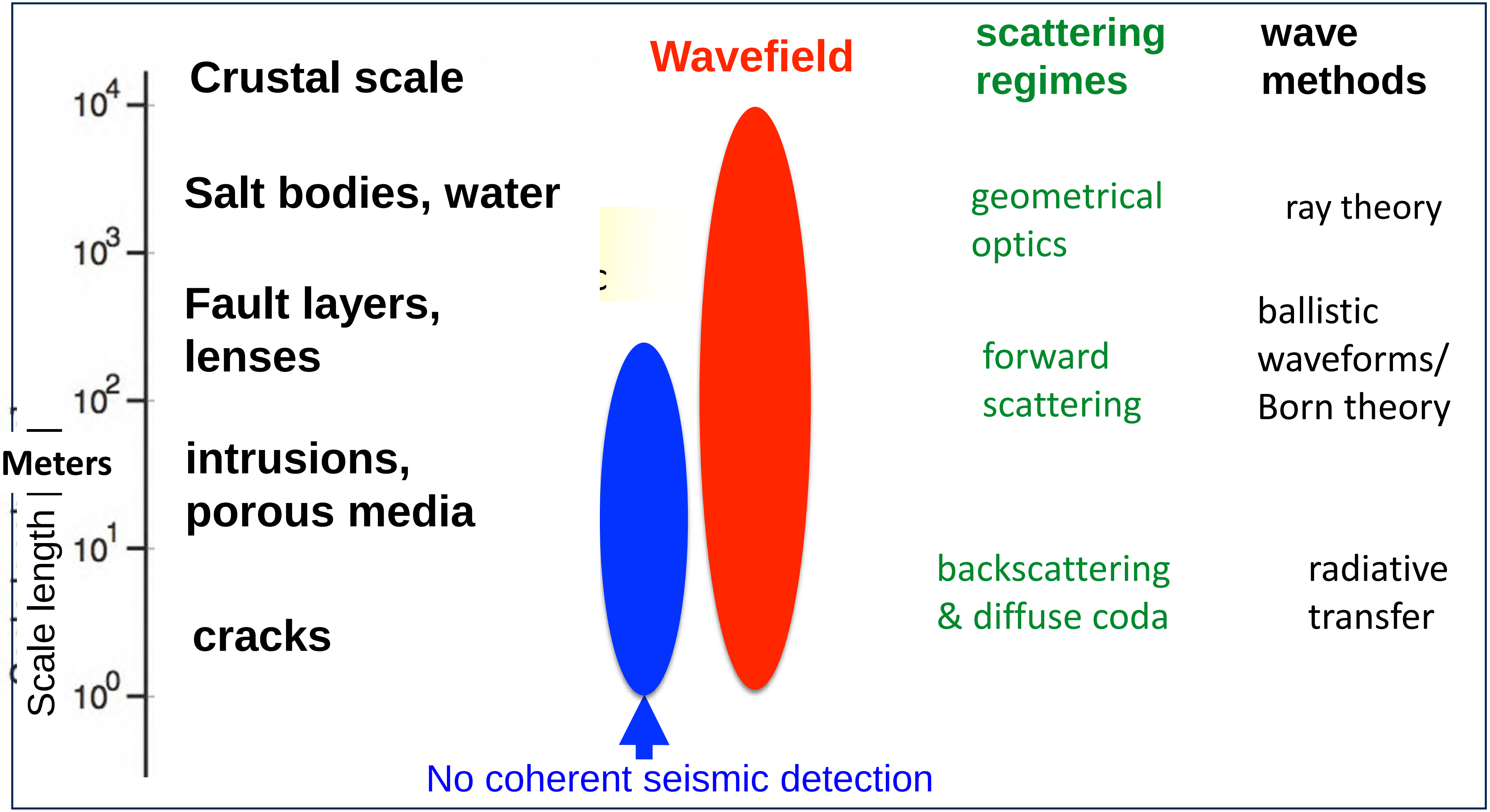
Moon, Mars, Earth & their digital twins



Born with memory loss

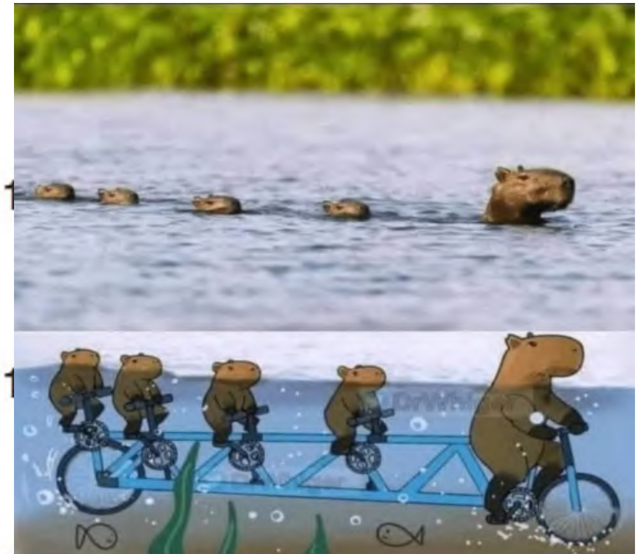


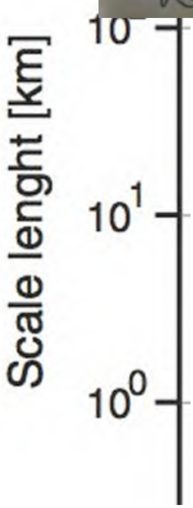
Why… Dark Earth matters



Shedding light on the darkness?

Incomplete data & model





Wavefield

Smoothies

Rough

Summary

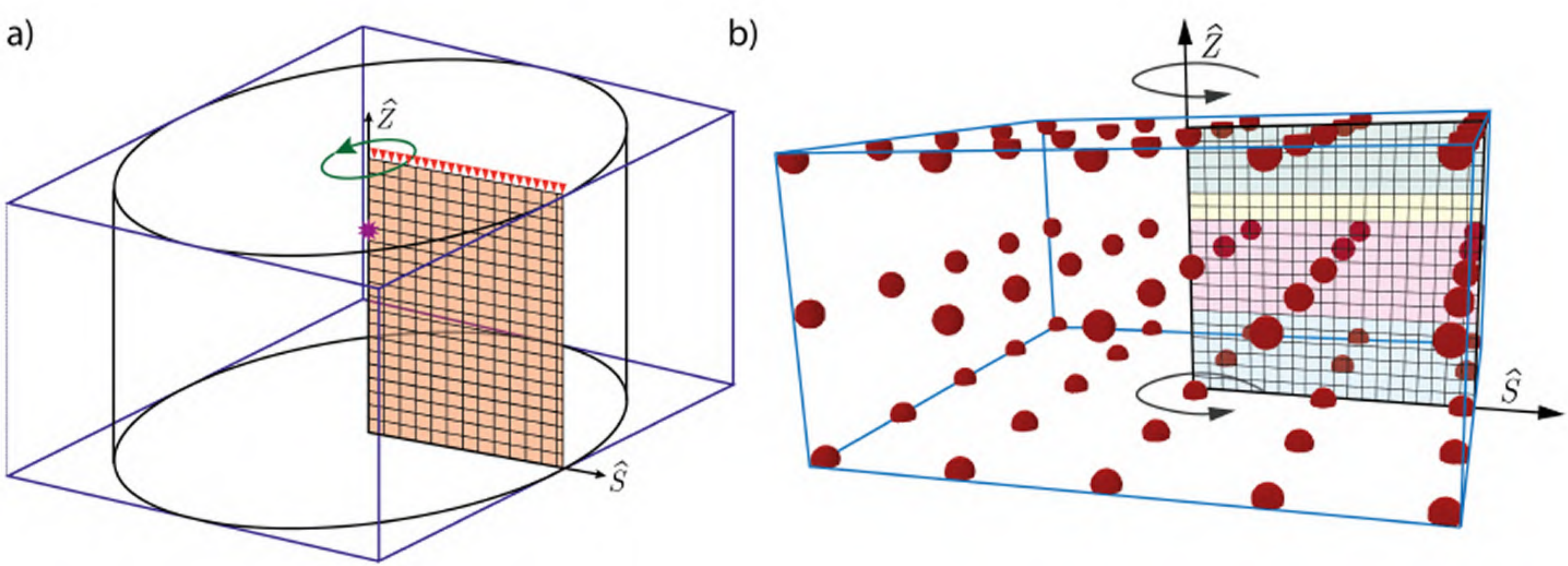
- (most/many/all?) wavefields have ballistic & diffusive components
- Simulating a parameter space is a MESS, always incomplete
- Quantifying complexity beyond phase/amplitude:
  - Coda Q, correlation matrices, multiscale entropy
  - MSE captures gradual transition between end-members
  - Indicative sensitivity for character of scattering objects: volume, strength, surface area… basis for inverse problem?
- Earth, Mars, Moon, PS MSE complexity at comparable scale
- Martian P/S coda have different levels of diffusivity
- Martian waveforms fairly complex, partially diffusive?
- Transitional regime may appeal to crucial questions in planetary interiors!

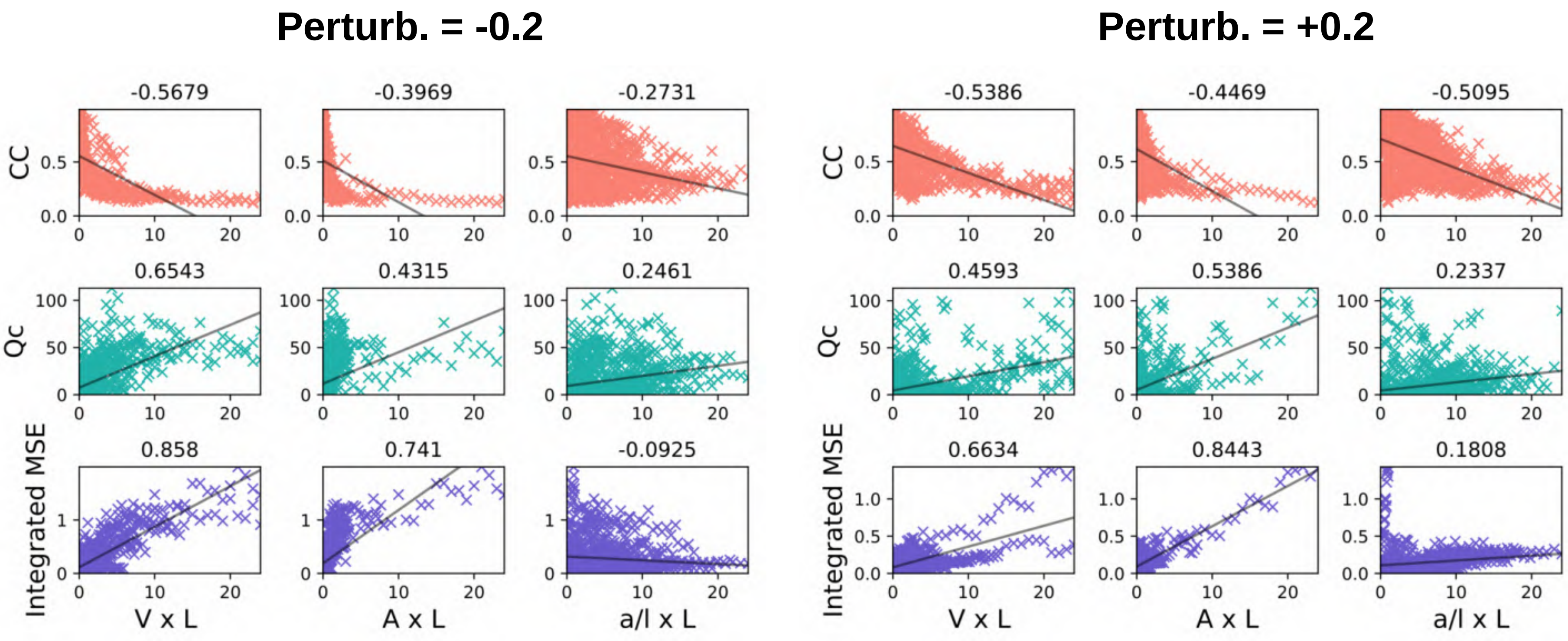
Modelling across scattering regimes

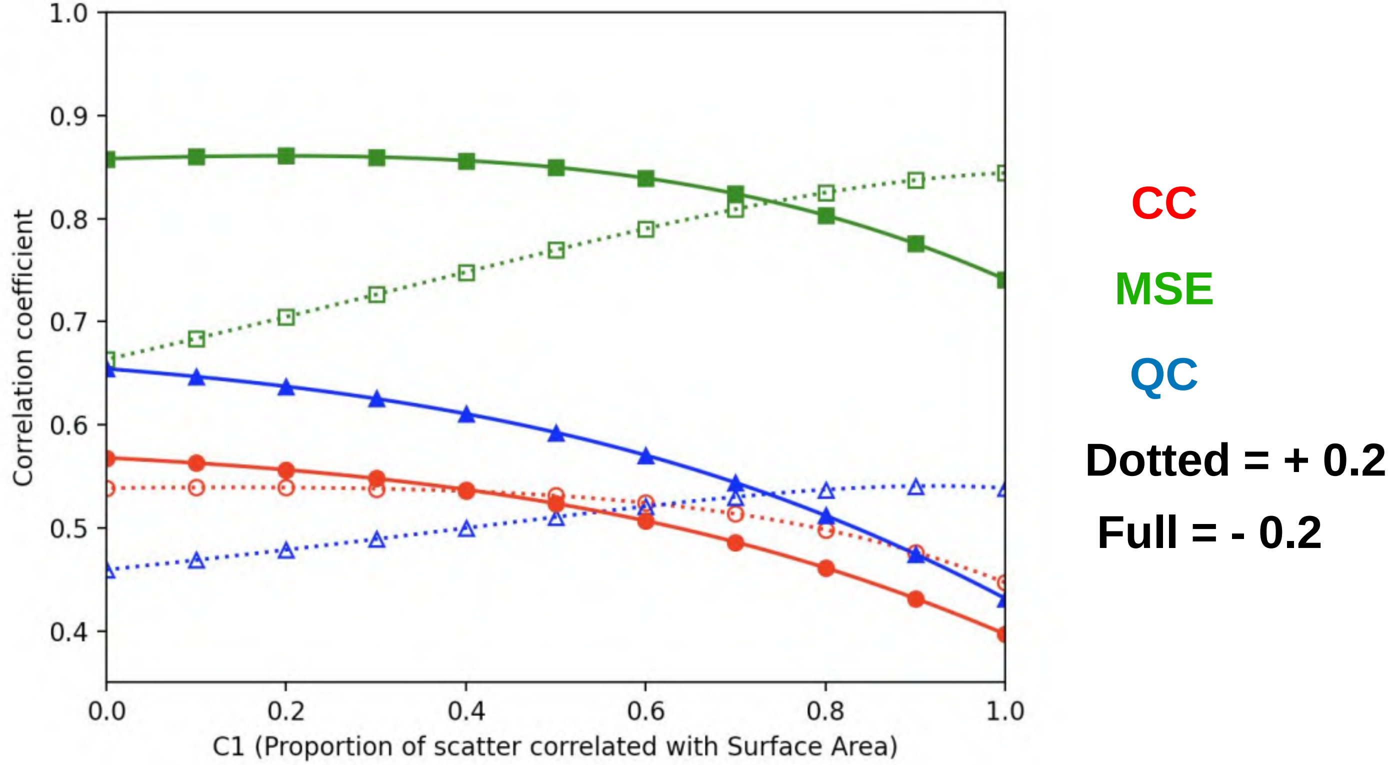
Spherical 'scatterers' of perturbed velocity/density

Homogenous 3D background model

Wave propagation using AxiSEM3D

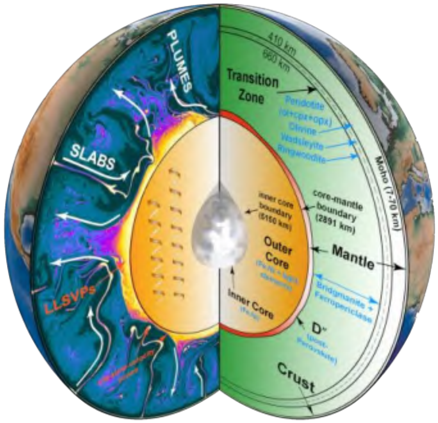


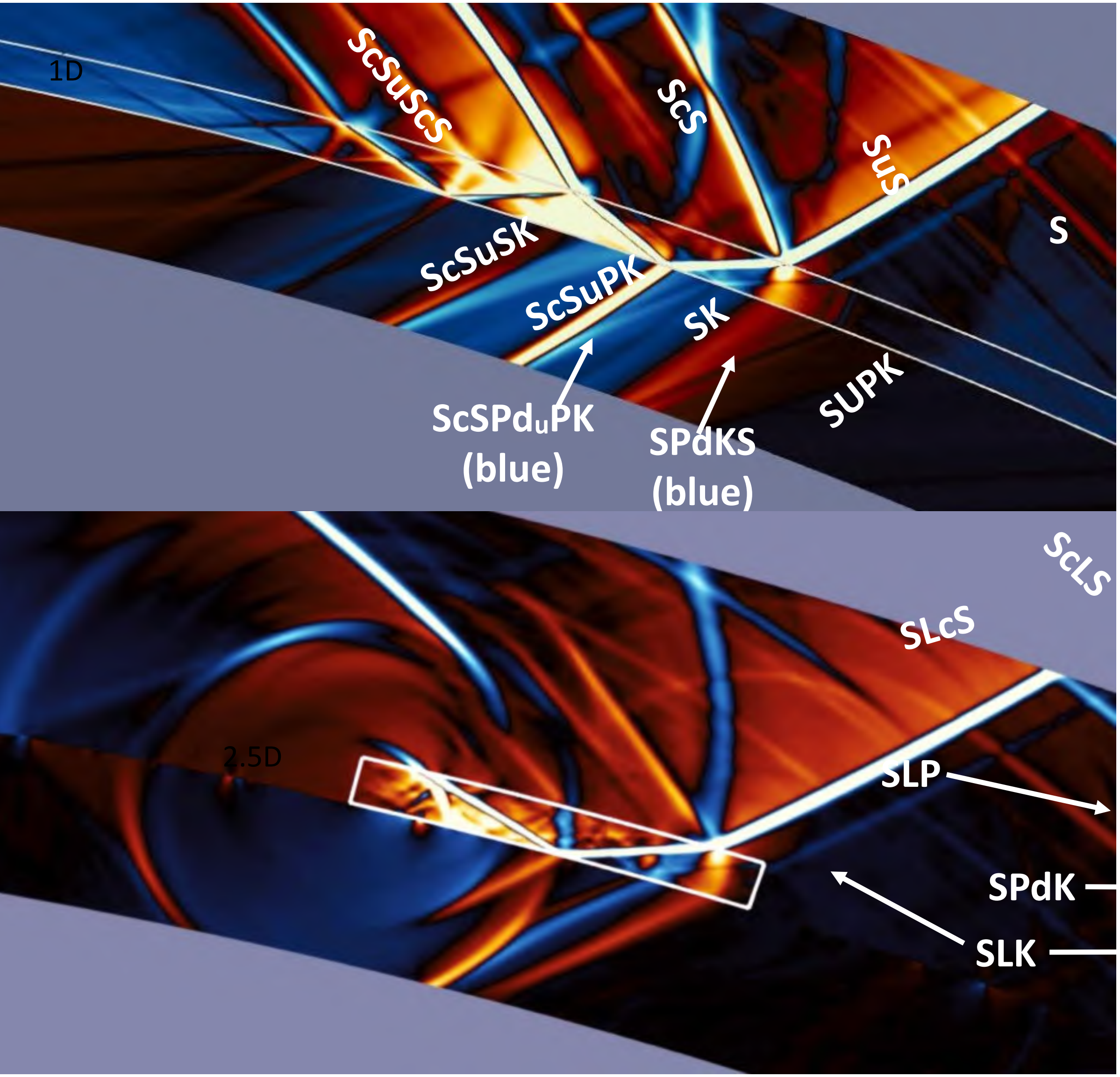

Perturb. = -0.2
Perturb. = +0.2
-0.5679
-0.3969
-0.2731
-0.5386
-0.4469
-0.5095
CC
0.6543
0.4315
0.2461
0.4593
0.5386
0.2337
Qc
0.858
0.741
-0.0925
0.6634
0.8443
0.1808
Integrated MSE
V x L
A x L
a/l x L
V x L
A x L
a/l x L



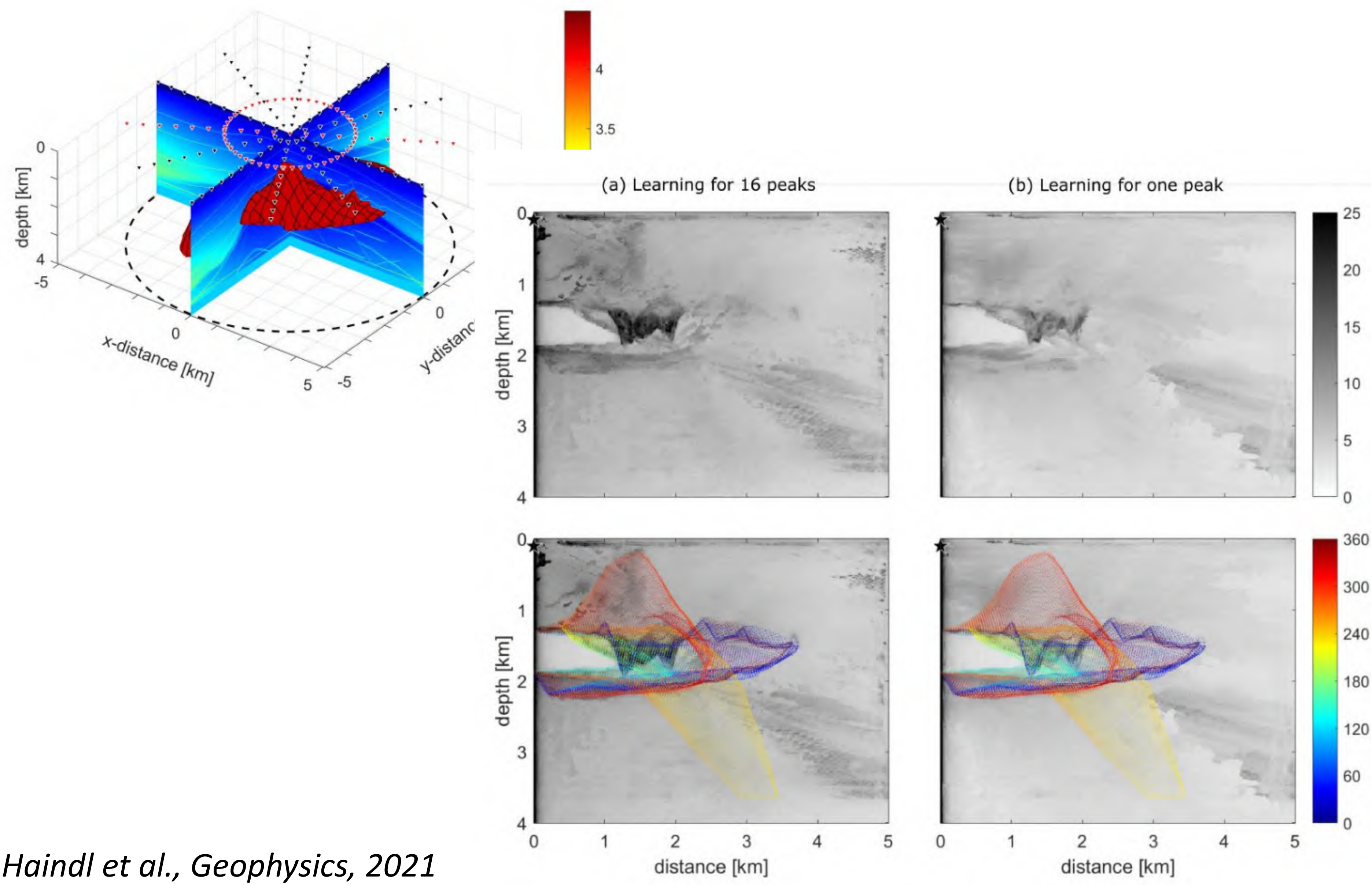
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Correlation coefficient
0.0
0.2
0.4
0.6
0.8
1.0
C1 (Proportion of scatter correlated with Surface Area)
CC
MSE
QC
Dotted = + 0.2
Full = - 0.2

How to decipher wavefield, 1D, 2D, 3D effects?



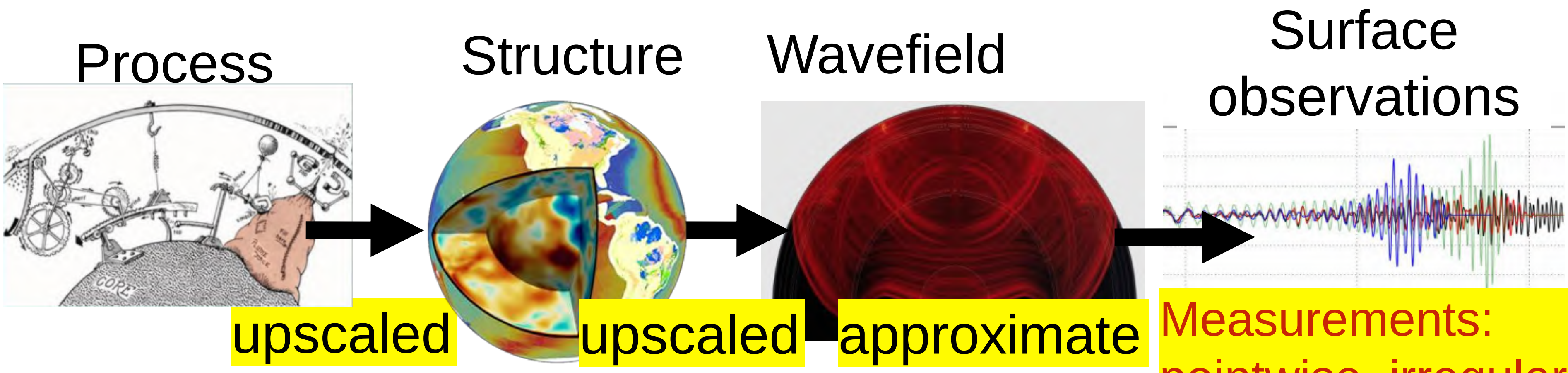


Azimuthal complexity adaptation: offshore salt bodies



Haindl et al., Geophysics, 2021

The inference/illumination/downscaling problem



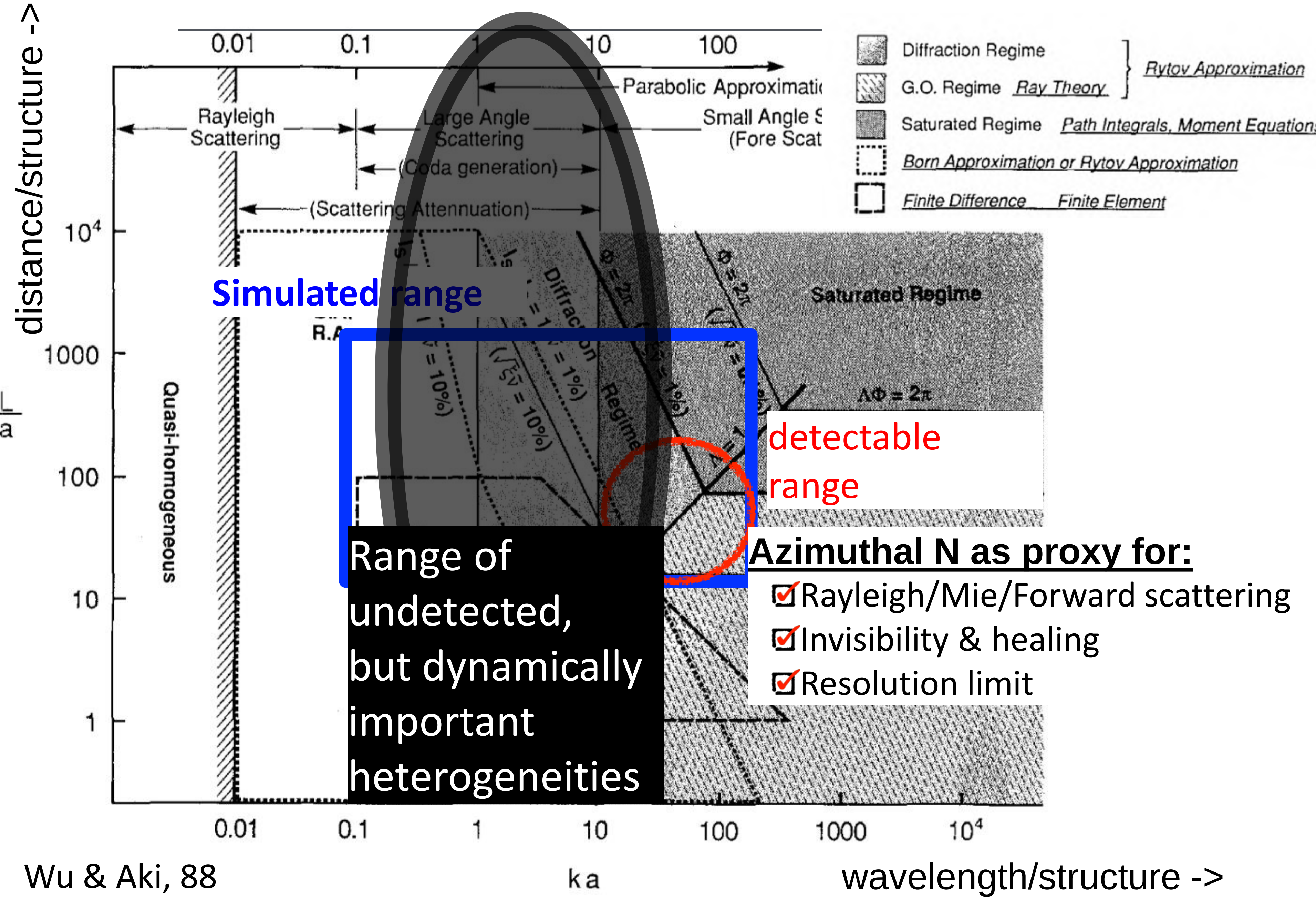
Measurements: pointwise, irregular subjective

Interpretation: Blurred, uncertain, unverifiable

Credit: @DrWhiger

Deterministic inference: ill-posed, non-unique, expensive

Scattering regimes & invisible structures



Wu & Aki, 88