Laboratory-Scale Investigation of Intact and Fractured Rocks’ Nonlinear Elastic Behavior *under Different Stress, Saturation and RH Conditions*

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U.S. DEPARTMENT OF ENERGY | Office of Science

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SPIN Workshop – May 2022

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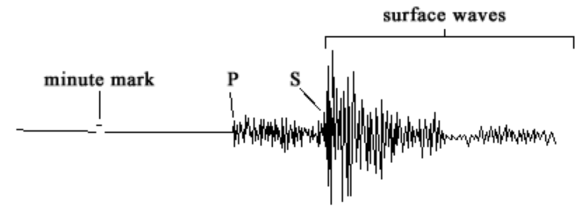
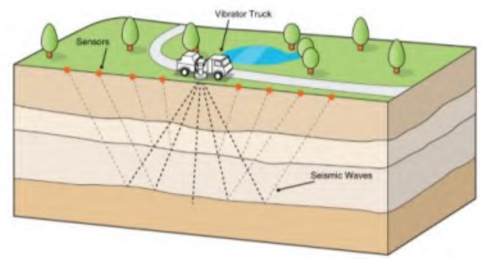
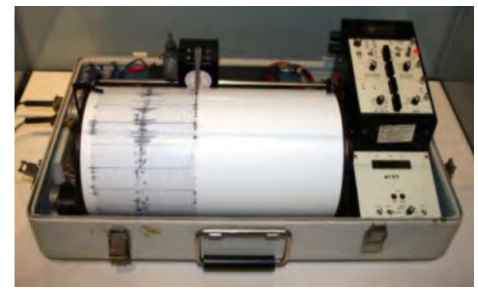
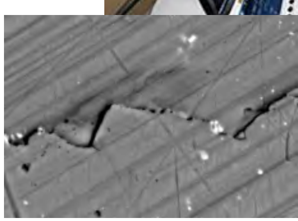
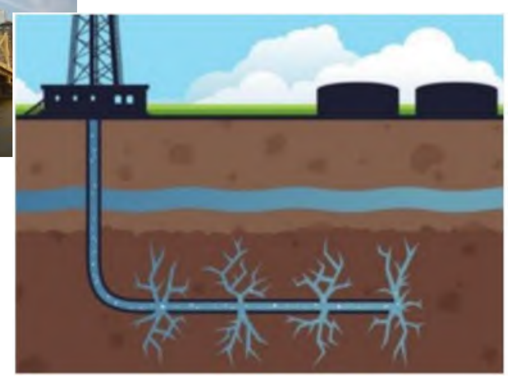
Exploiting the nonlinear elastic properties has implications in numerous fields across length scales











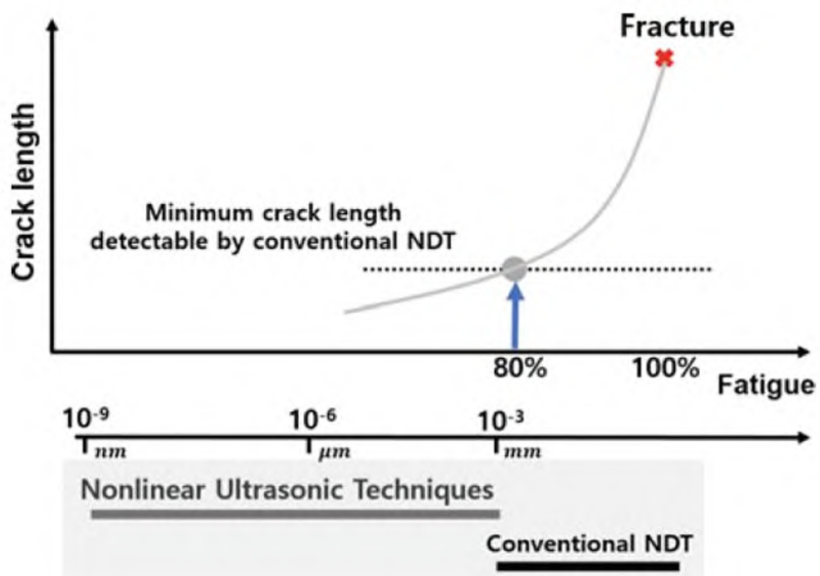
µm km

Ultrasonic (kHz) Seismic (0.1Hz)

Image source: P. Lundgren, Nature, 2014; wikipedia

Nonlinear elastic response can reveal incipient damage and characterize a wide range of materials

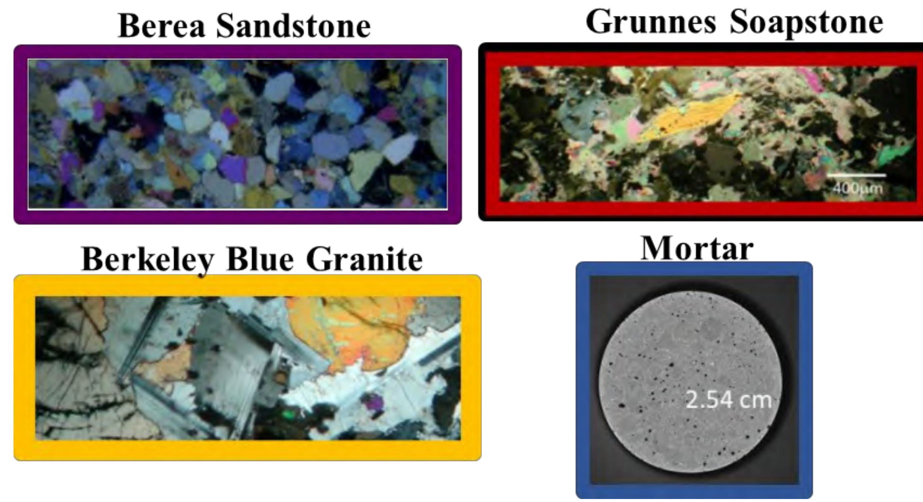
Early Damage detection

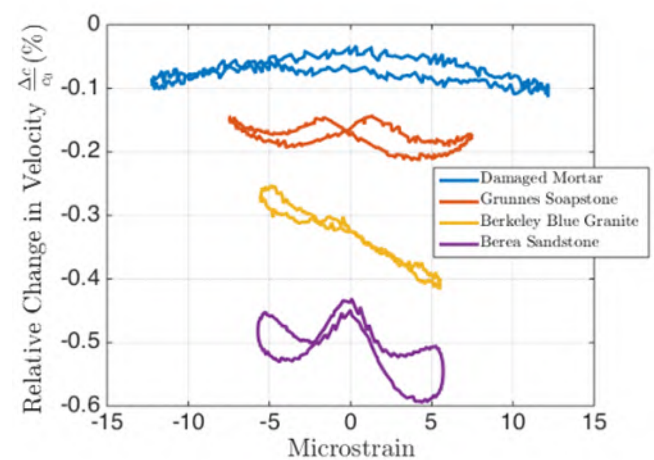


Jhang, Kyung-Young, *et al*. ***Measurement of Nonlinear Ultrasonic Characteristics***. Springer, 2020.

Jin et al. *JONDE* (2017)
Shokouhi et al. *Ultrasonics* (2017)
Jin et al. *JAP* (2018)
Jin et al. *JONDE* (2019)
Bozek et al. *NDT&E* (2021)
…

Material Characterization





Riviere, Shokouhi, Guyer & Johnson *JGR* (2015)

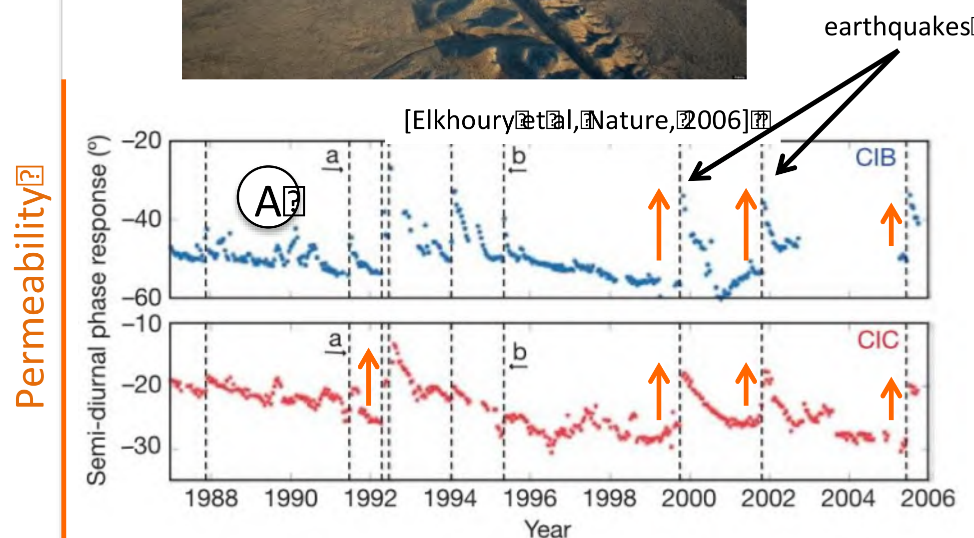
Correspondence between nonlinear elasticity and permeability change?

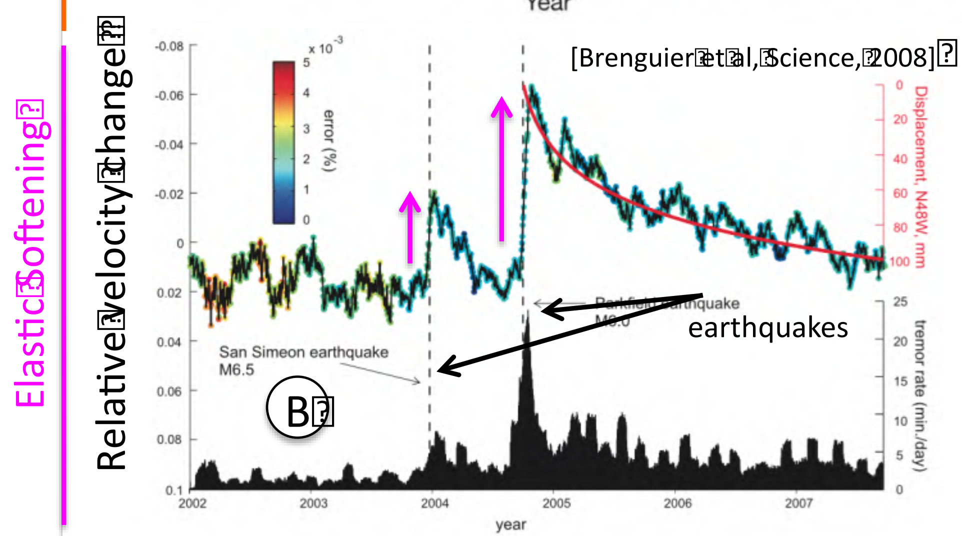
Shokouhi et al. *GRL* (2020)

permeability transients caused by seismic waves from distant earthquakes

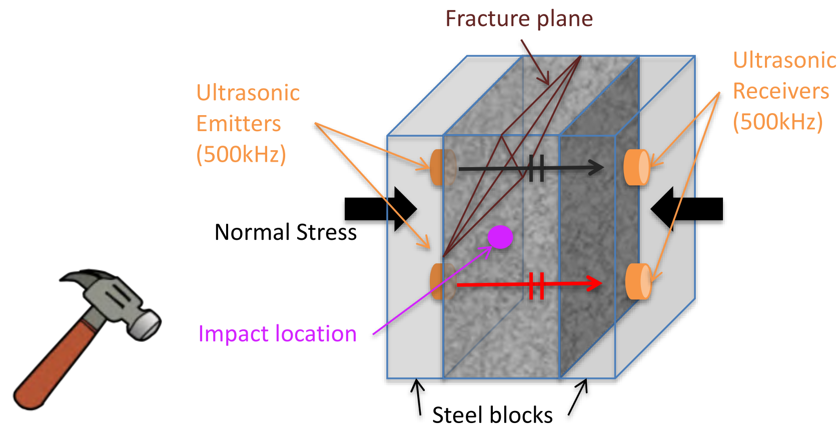
co-seismic elastic softening & post-seismic relaxation

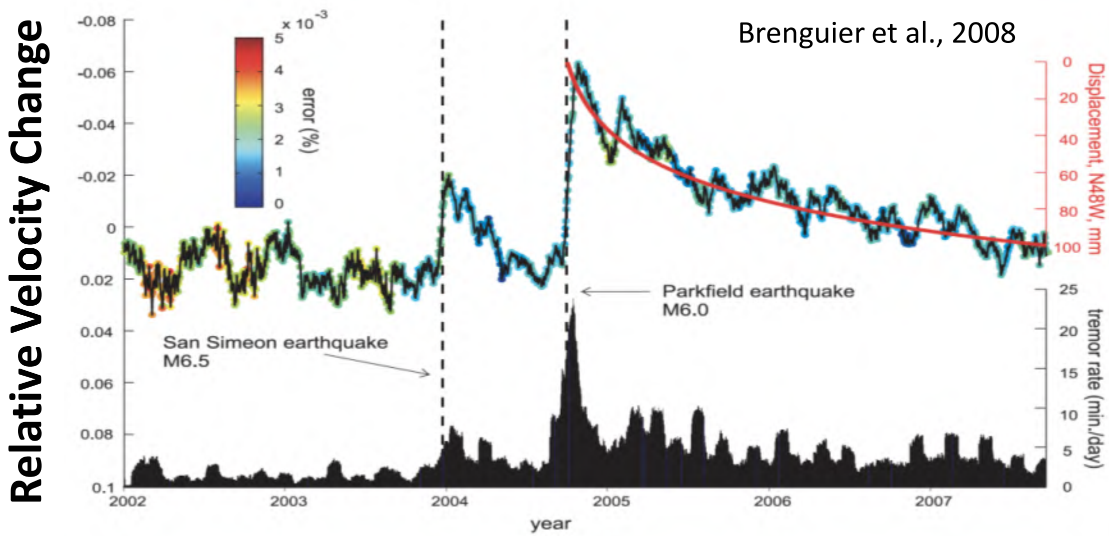
Field-scale

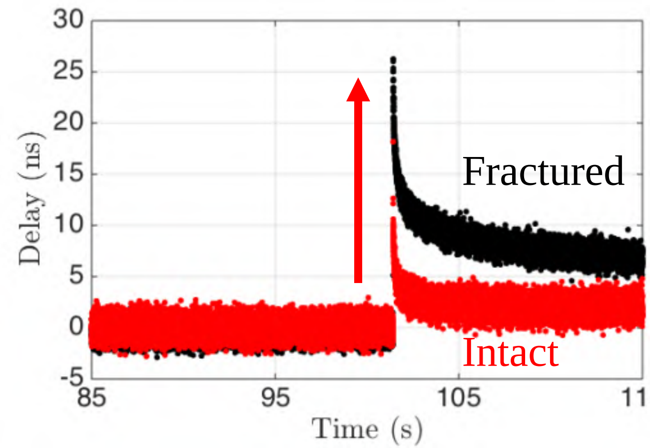




Elastic softening and relaxation phenomena are observed in both field and laboratory scales



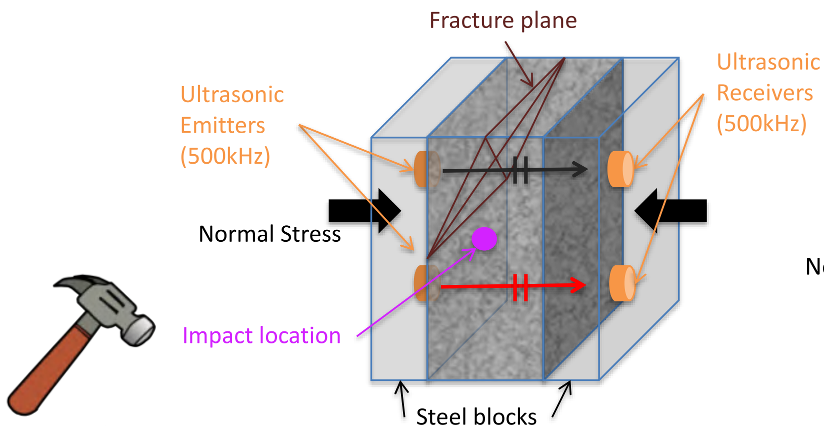


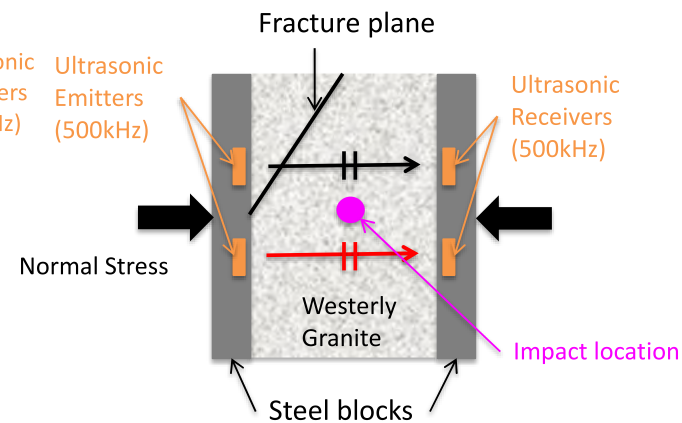


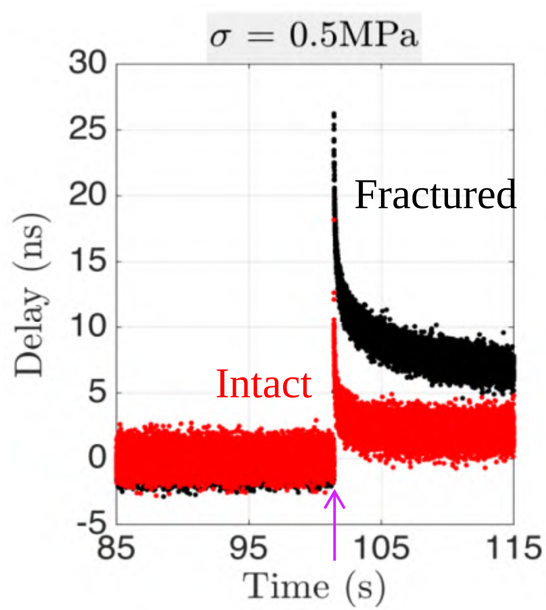
Earth Scale

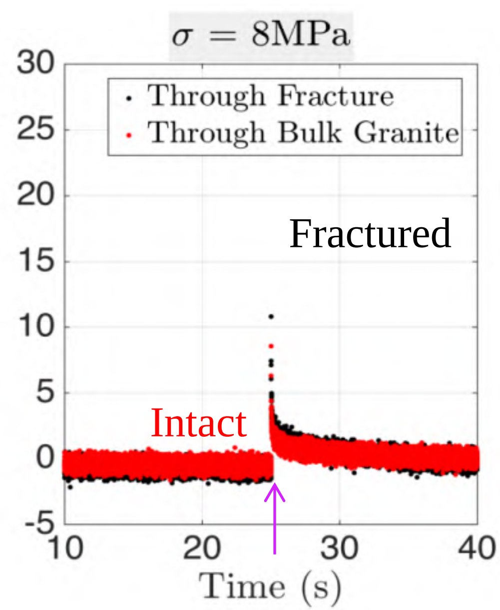
Laboratory Scale

Nonlinear elastic response depends on stress and fracture state *(as well as RH and saturation)*

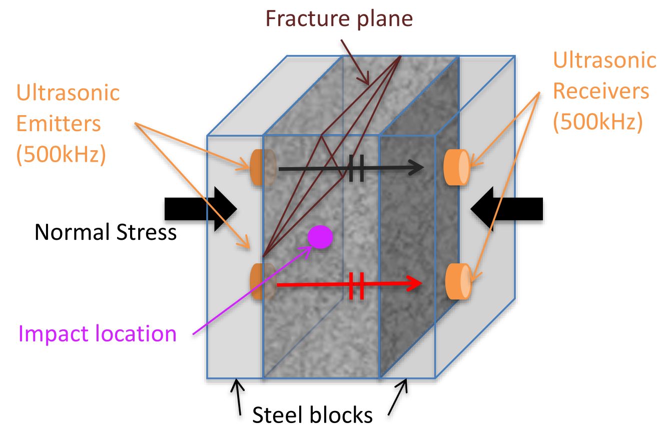


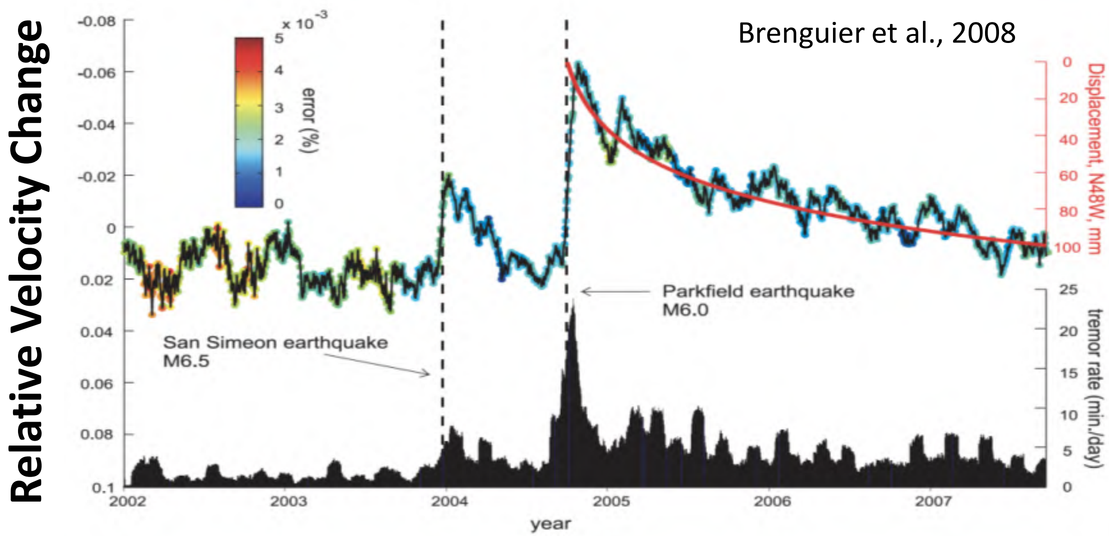




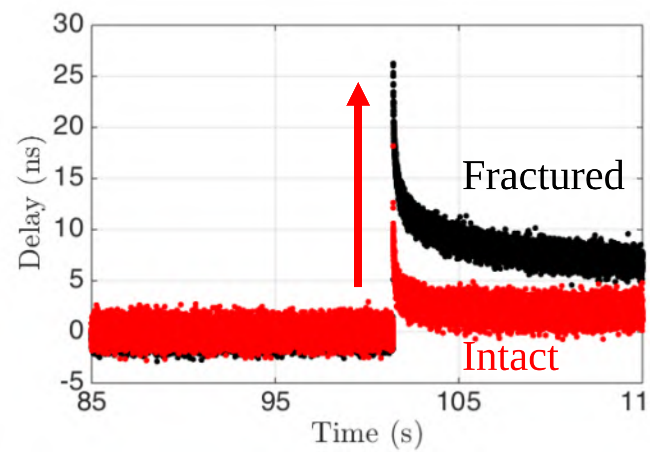


To understand the phenomenon at the Earth scale, we study nonlinear elasticity at the laboratory scale



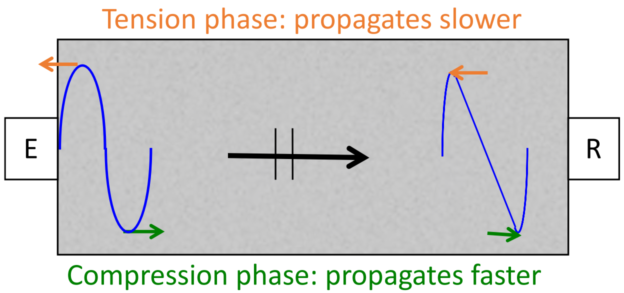


Earth Scale



Laboratory Scale

Outline



Principles:

- Linear *vs* Nonlinear Wave propagation
- How to measure nonlinear elasticity?

Dynamic Acousto-Elastic Testing (DAET):

- How to do DAET?
- Intact rock characterization
- Fractured rock's response under in-situ stress and saturation

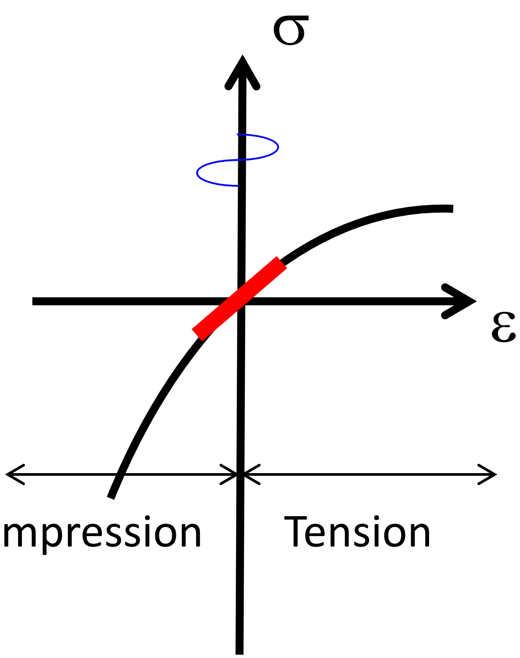
Outlook:

- New observations in granular media, Coupled X-ray CT and DAET experiments

Linear Acoustics/Ultrasound

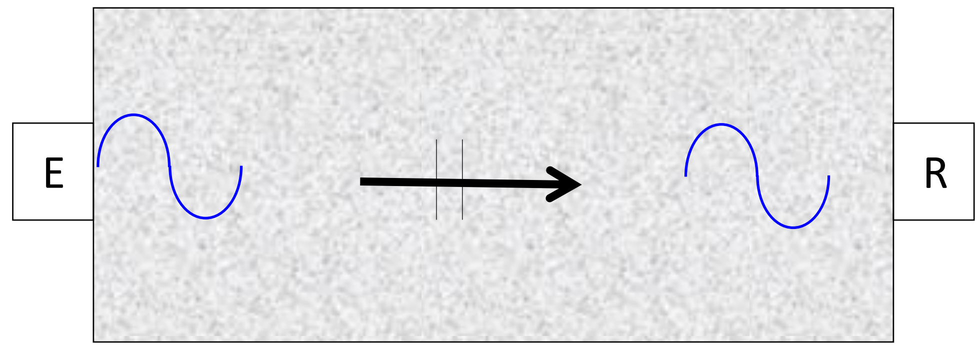
Small-amplitude wave: linear wave propagation

$$s = M(e)e$$



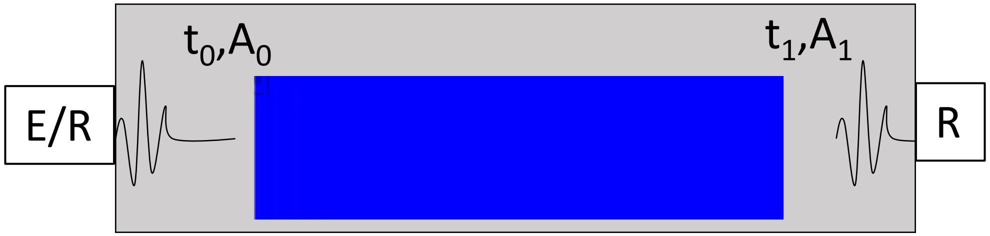
$$M(e) » M_0$$

$$s = M_0 e$$

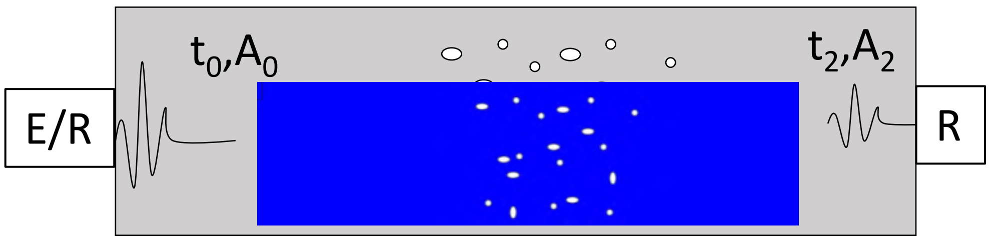


Speed of sound: $c_0 = \sqrt{\dfrac{M_0}{r}}$

σ : stress
ε : strain
M_0 : linear elastic modulus
M : nonlinear elastic modulus

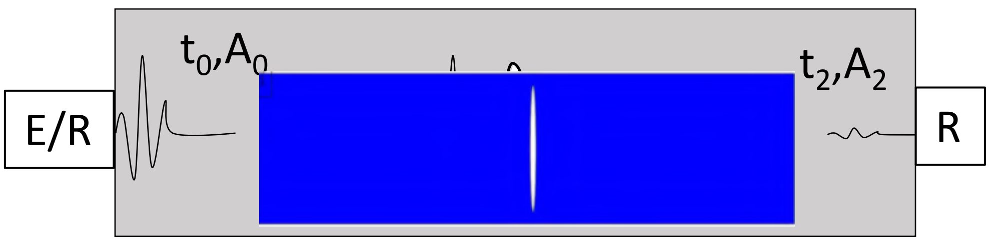


t = Arrival time -> velocity
A = Amplitude -> attenuation



$t_2 > t_1$ and $A_2 < A_1$

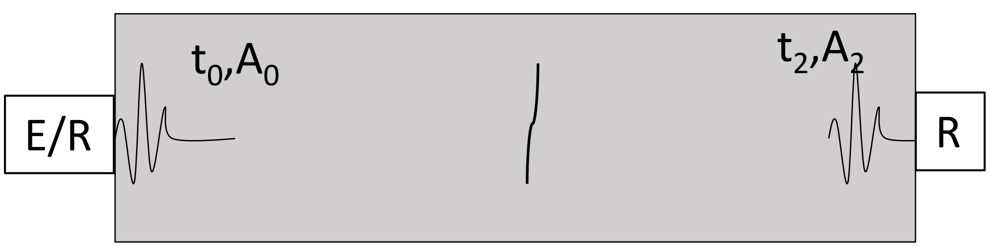
Slower velocity + Lower amplitude



$t_2 > t_1$ and $A_2 << A_1$

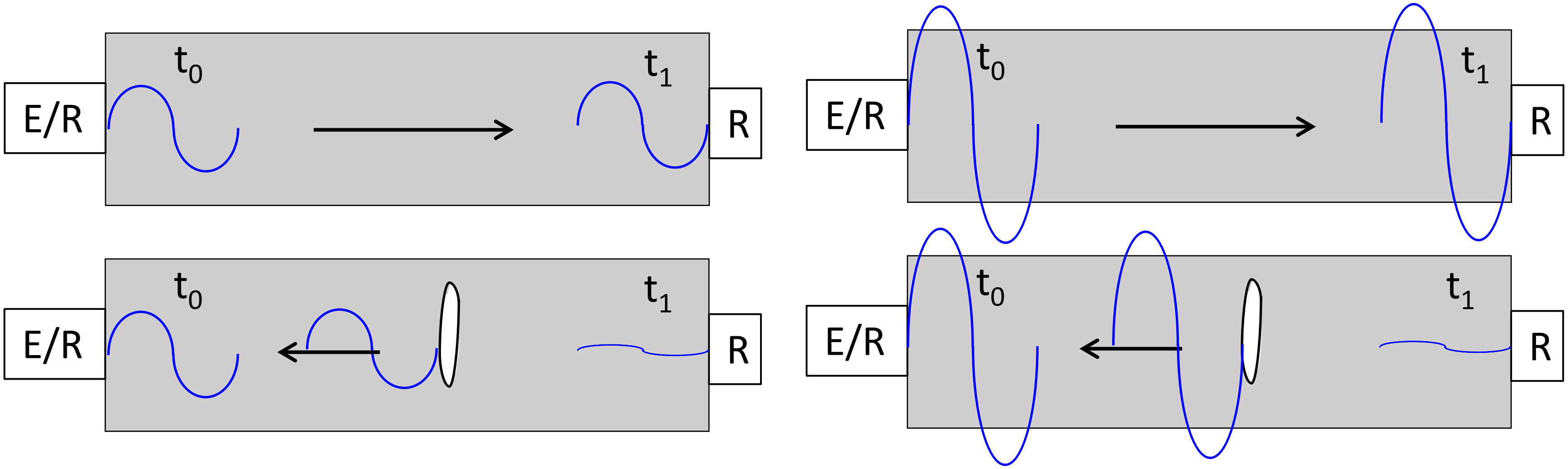
Reflection

Slower velocity + Lower amplitude

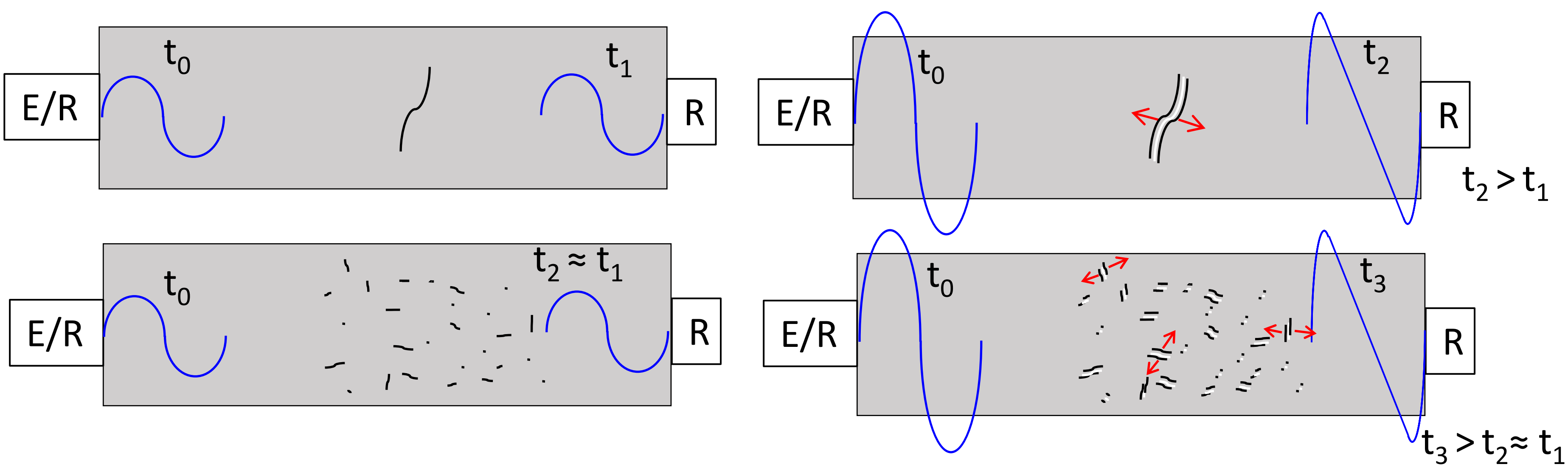


$t_2 \approx t_1$ and $A_2 \approx A_1$

Closed cracks are “invisible”



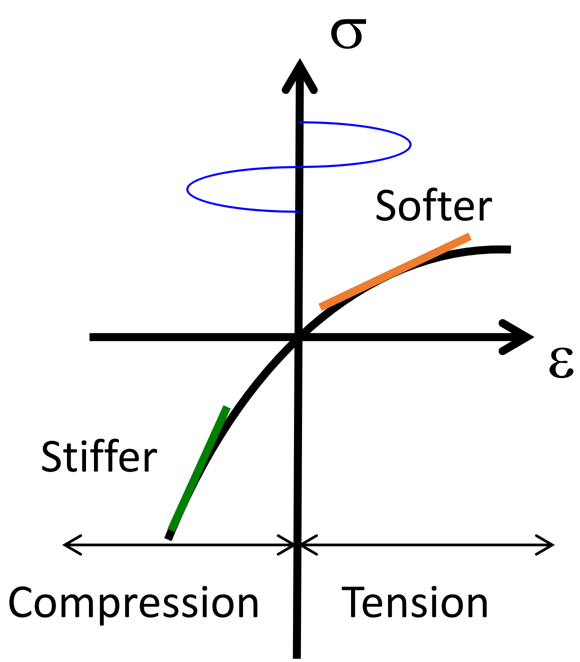
Small-amplitude *vs* Large-amplitude (Linear *vs* Nonlinear)



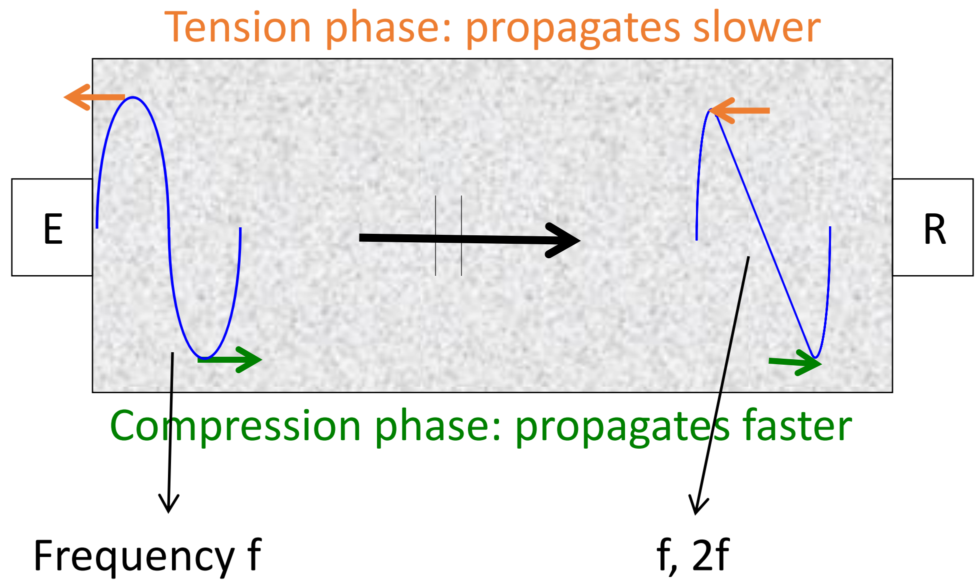
Nonlinear Acoustics/Ultrasound

Large-amplitude wave: nonlinear wave propagation

$$s = M(e)e$$



$$M(e) = M_0(1 - be)$$

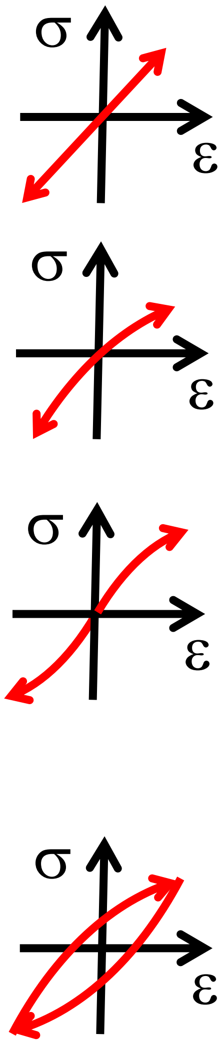


σ : stress
ε : strain
M_0 : linear elastic modulus
M : nonlinear elastic modulus
β : nonlinear coefficient

$$b \mu \frac{A_{2f}}{A_f^2}$$

β is related to third-order elastic constants C_{IJK}
(second order constants are C_{IJ})

Implications of nonlinear elasticity in ultrasonic testing



$$M(e) = M_0$$ Linear

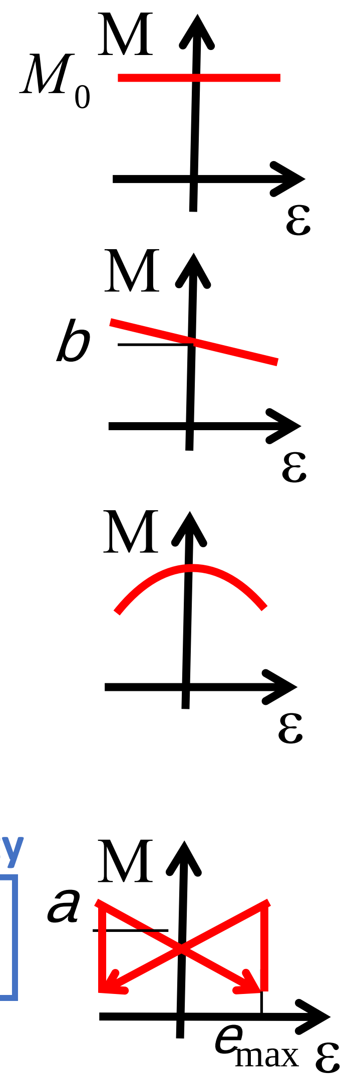
$$M(e) = M_0(1 - be)$$

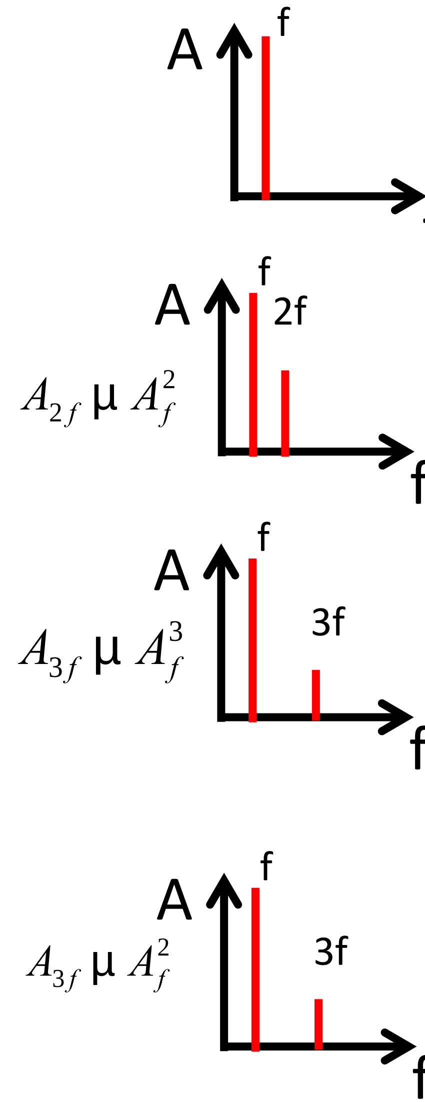
Classical nonlinearity

$$M(e) = M_0(1 - de^2)$$

Hysteretic (non-classical) nonlinearity

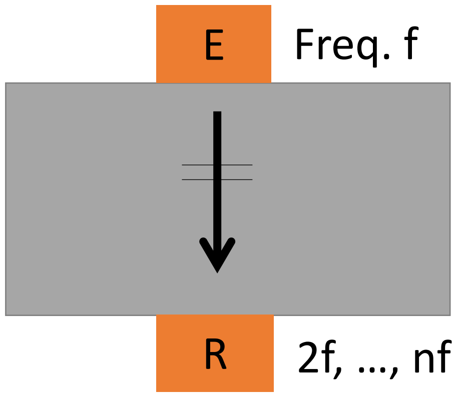
$$M(\varepsilon) = M_0(1 - \alpha(\varepsilon_{max} + \varepsilon sign(\dot{\varepsilon})))$$



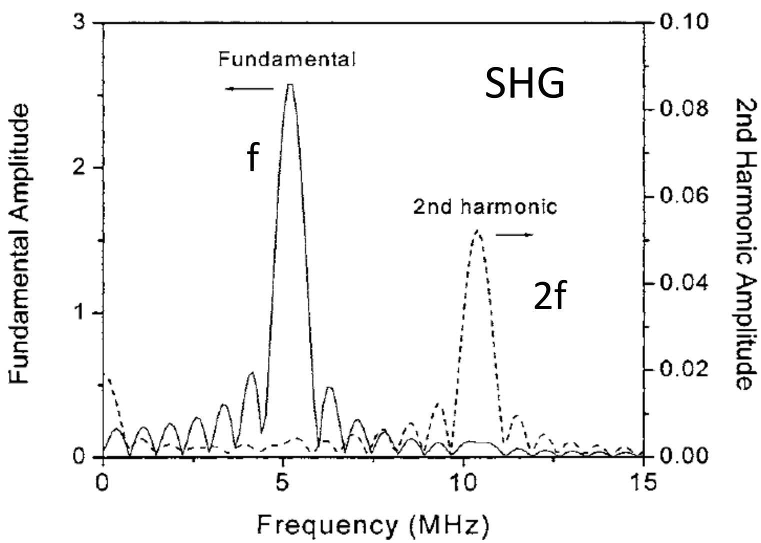


K.E.A. Van Den Abeele, et al, Res. in Nondestruct. Eval., 2000

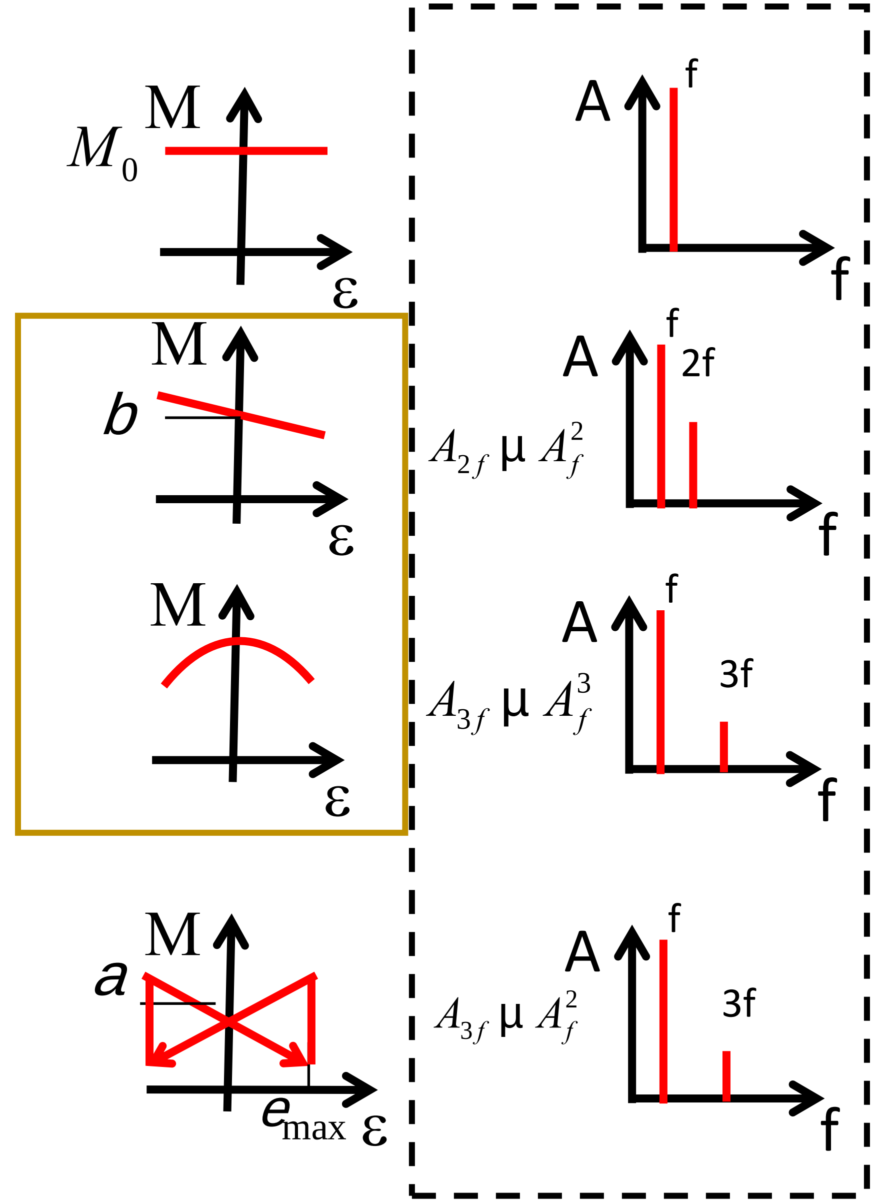
Amplitude of higher harmonics gives classical nonlinearity



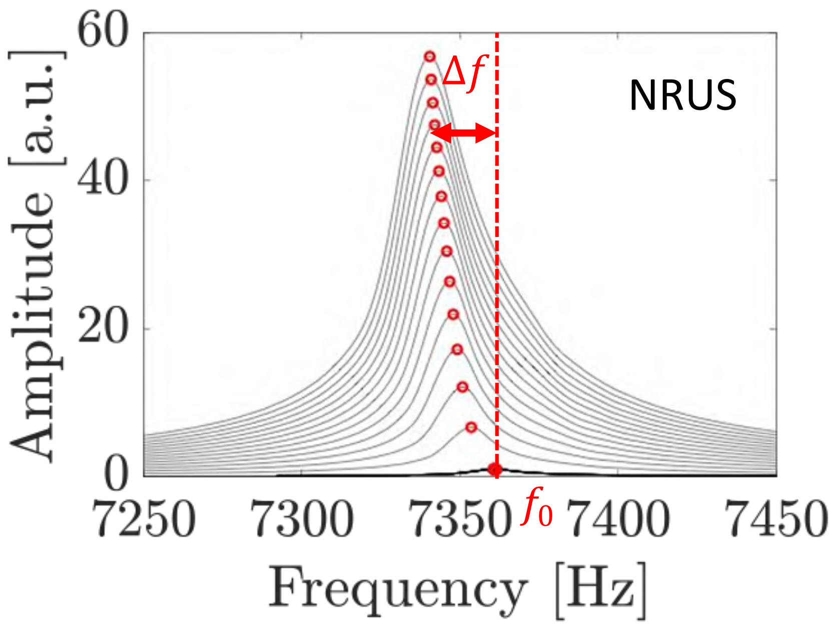
$$b \mu \frac{A_{2f}}{A_f^2}$$

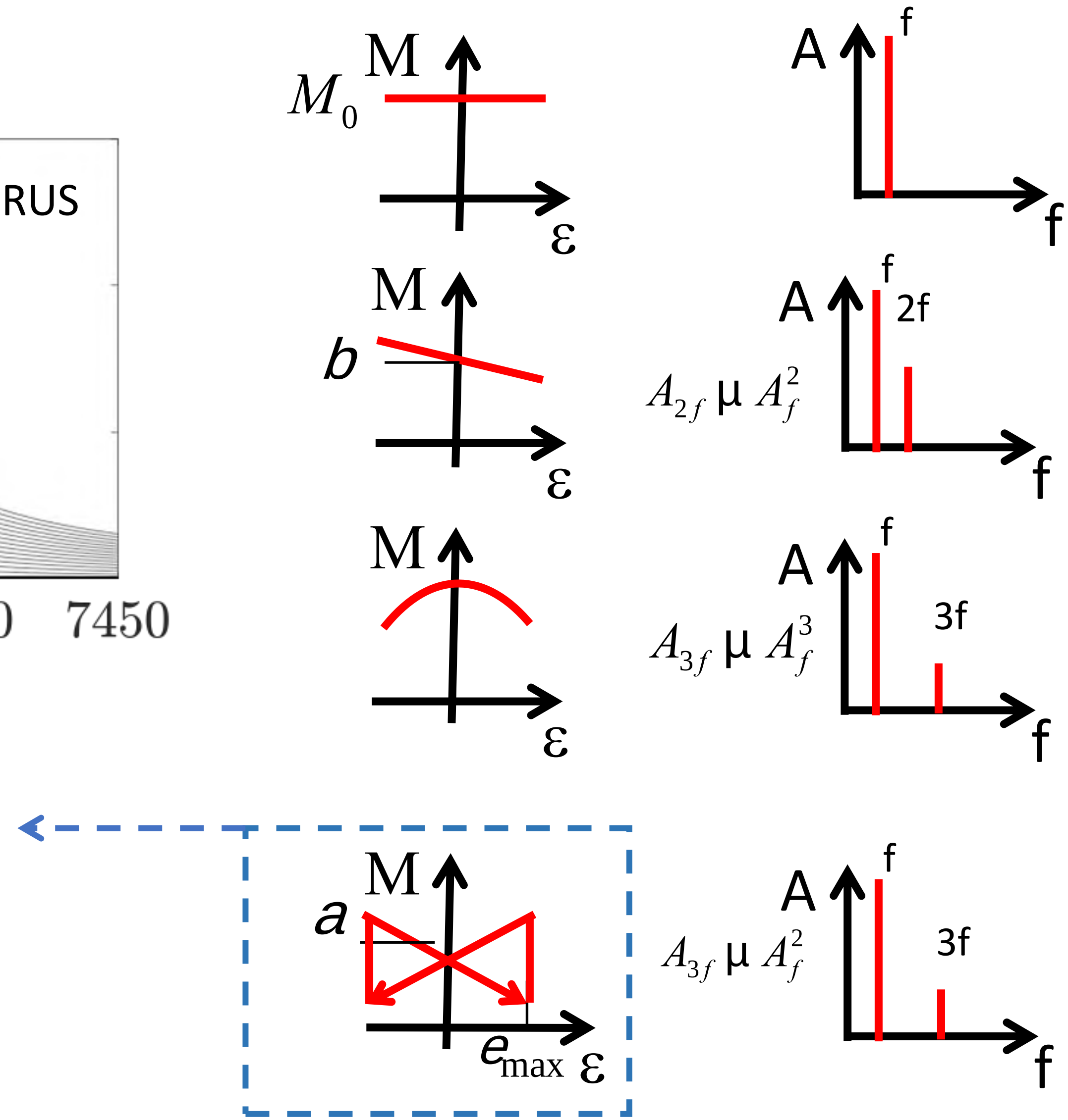


Kim et al., J. Acoust. Soc. Am, 2006



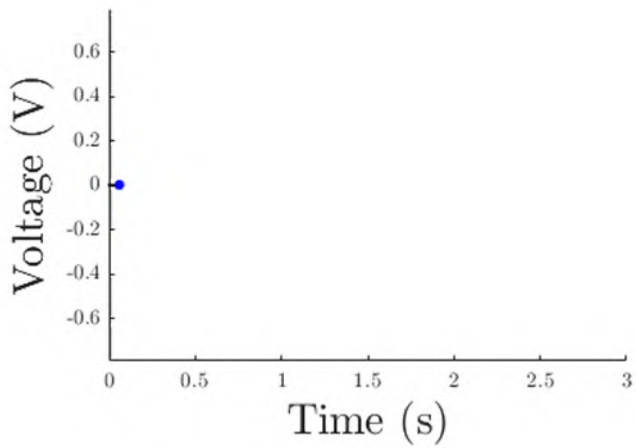
Shift in resonance frequency gives hysteretic nonlinearity

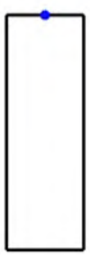




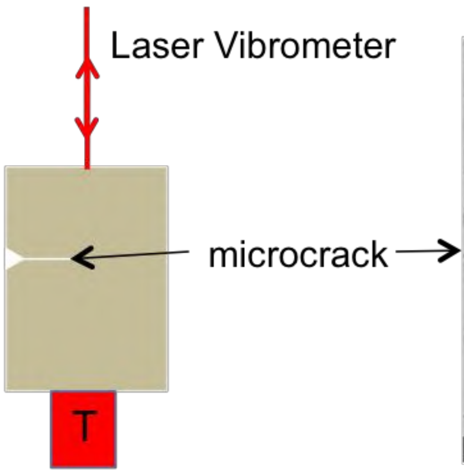
$$\frac{\Delta f}{f_0} = \alpha \, \varepsilon_{max}$$

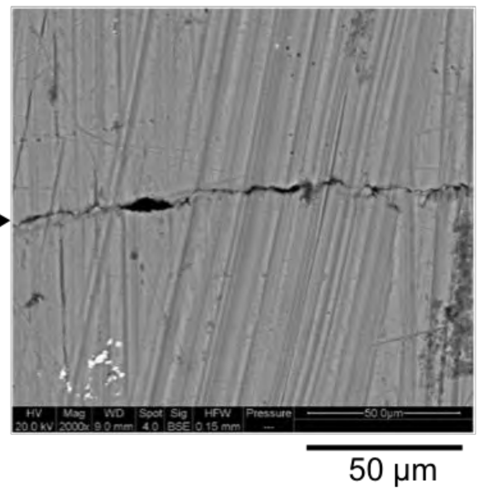
Nonlinear Resonant Ultrasound Spectroscopy (NRUS)

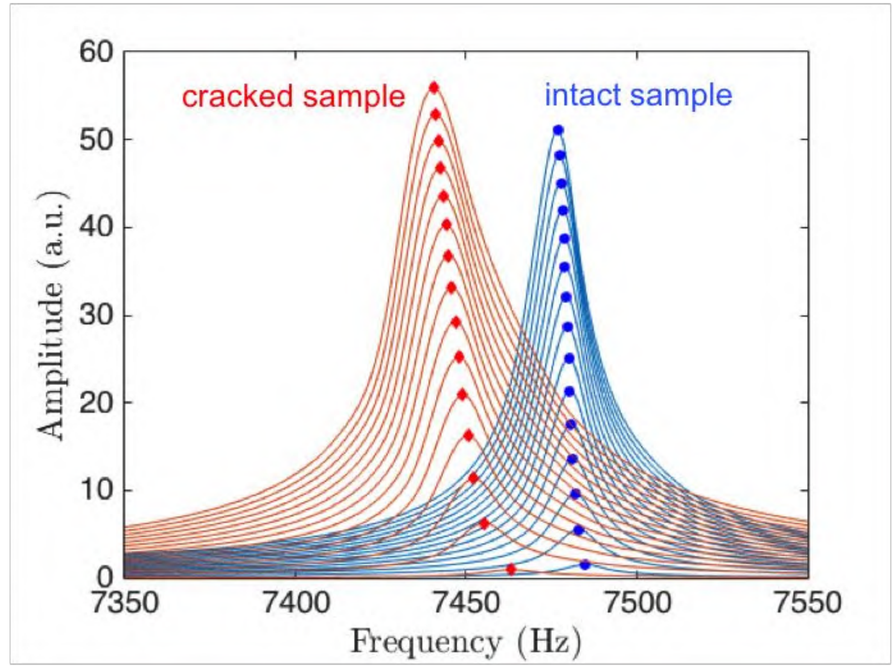




A larger hysteretic nonlinearity α is associated with more “damage”

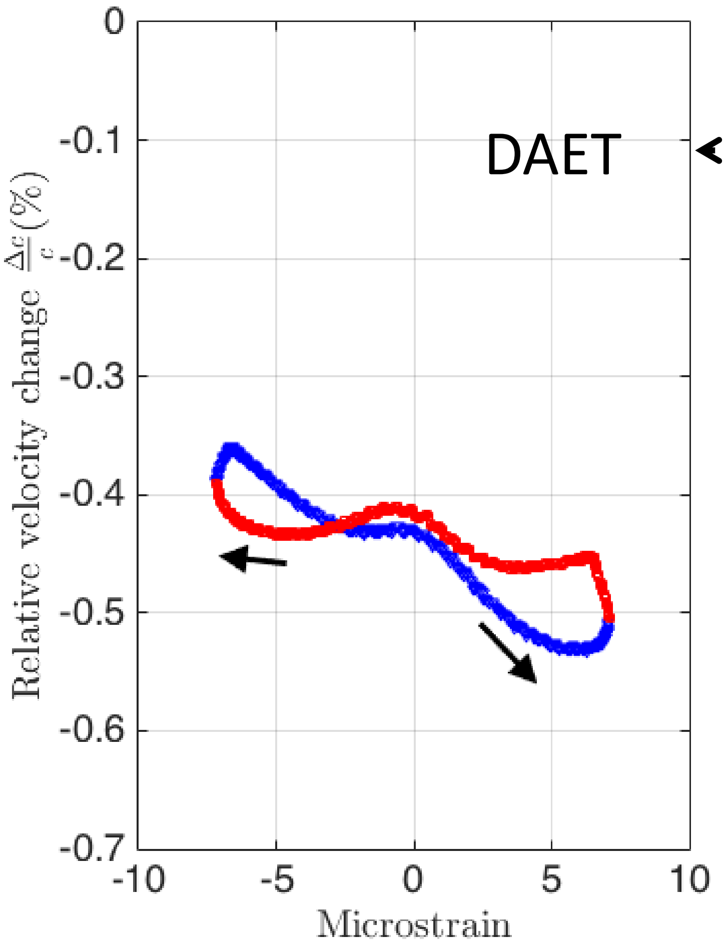


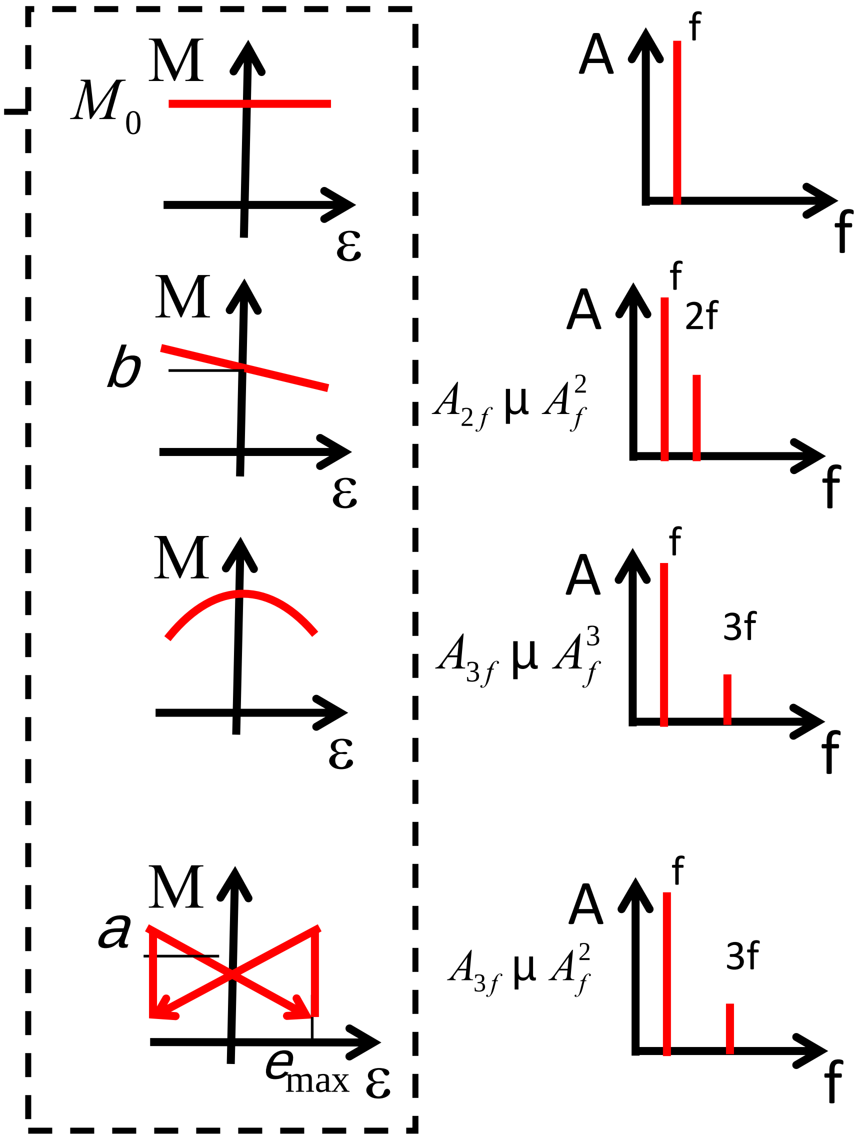




Sensitive to the presence of elongated, microscopic/closed defects

Could we measure to M *vs* strain directly?

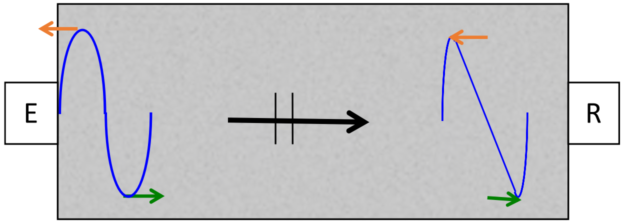




$$\frac{dM}{M} » 2\frac{dc}{c}$$

Direct observation of M = f(ε)

Outline



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Dynamic Acousto-Elastic Testing (DAET):

- How to do DAET?
- Intact rock characterization
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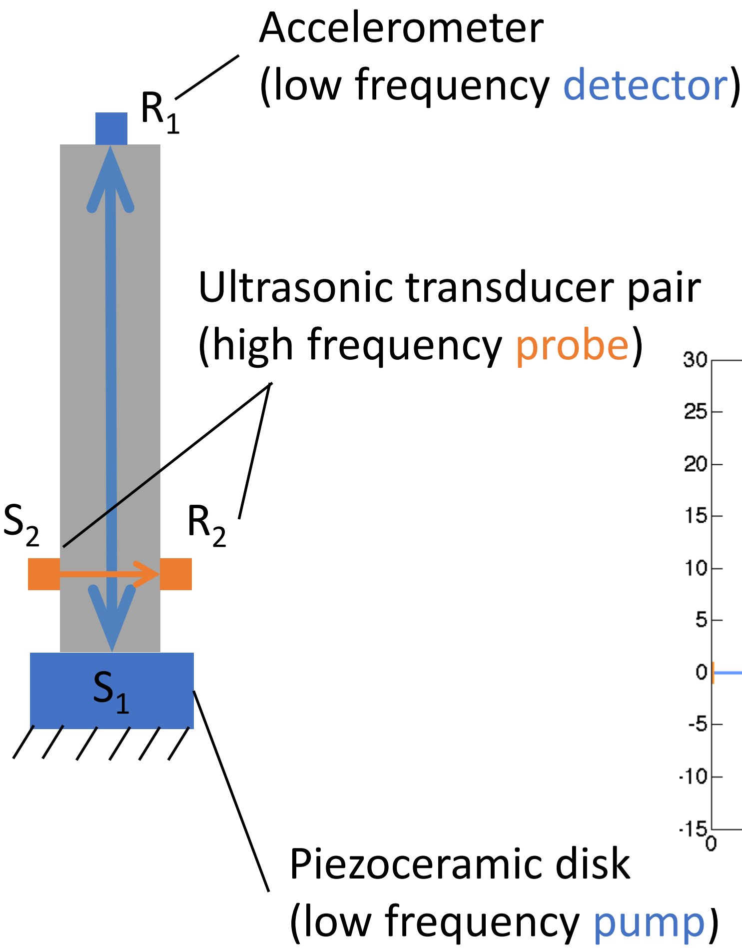
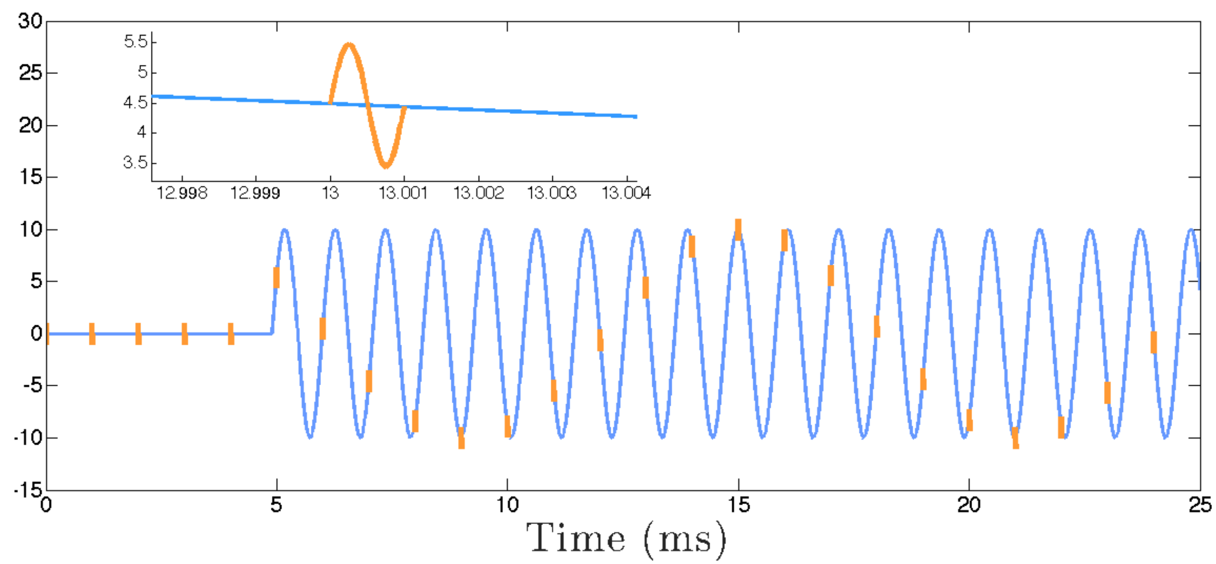
Outlook:

- New observations in granular media, Coupled X-ray CT and DAET experiments

Dynamic acousto-elastic testing (DAET) is a ‘pump’ and ‘probe’ approach

Renaud *et al.*, JASA, 2011 ; Rivière et al, JAP, 2013

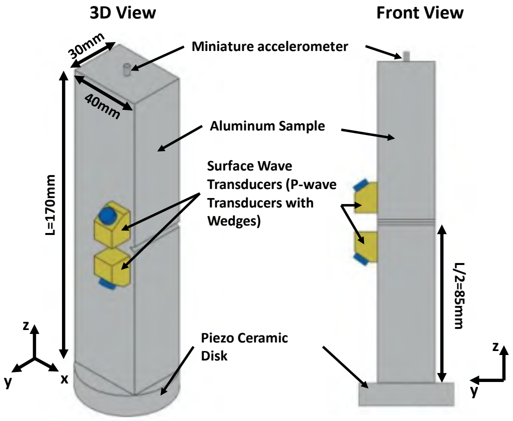




There are other DAET test configurations …

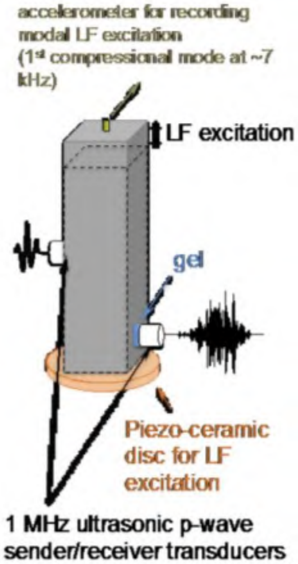
with surface wave probe

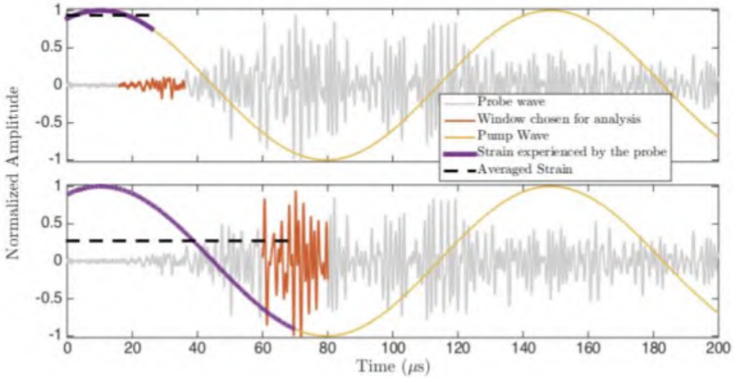
Jin et al. *JAP* (2018)
Jin et al. *JMPS* (2020)



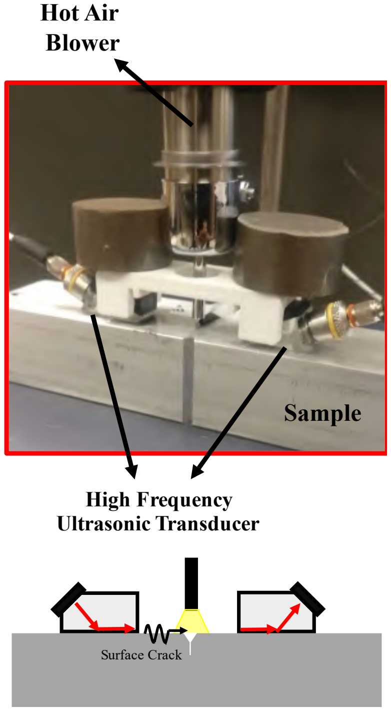
with coda wave probe

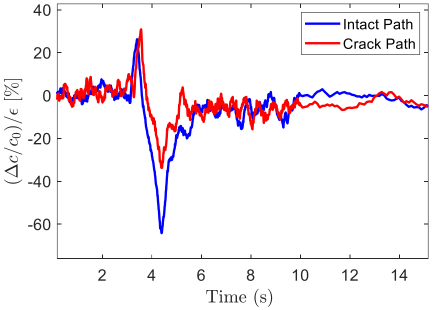
Shokouhi et al. *Ultrasonics* (2017)



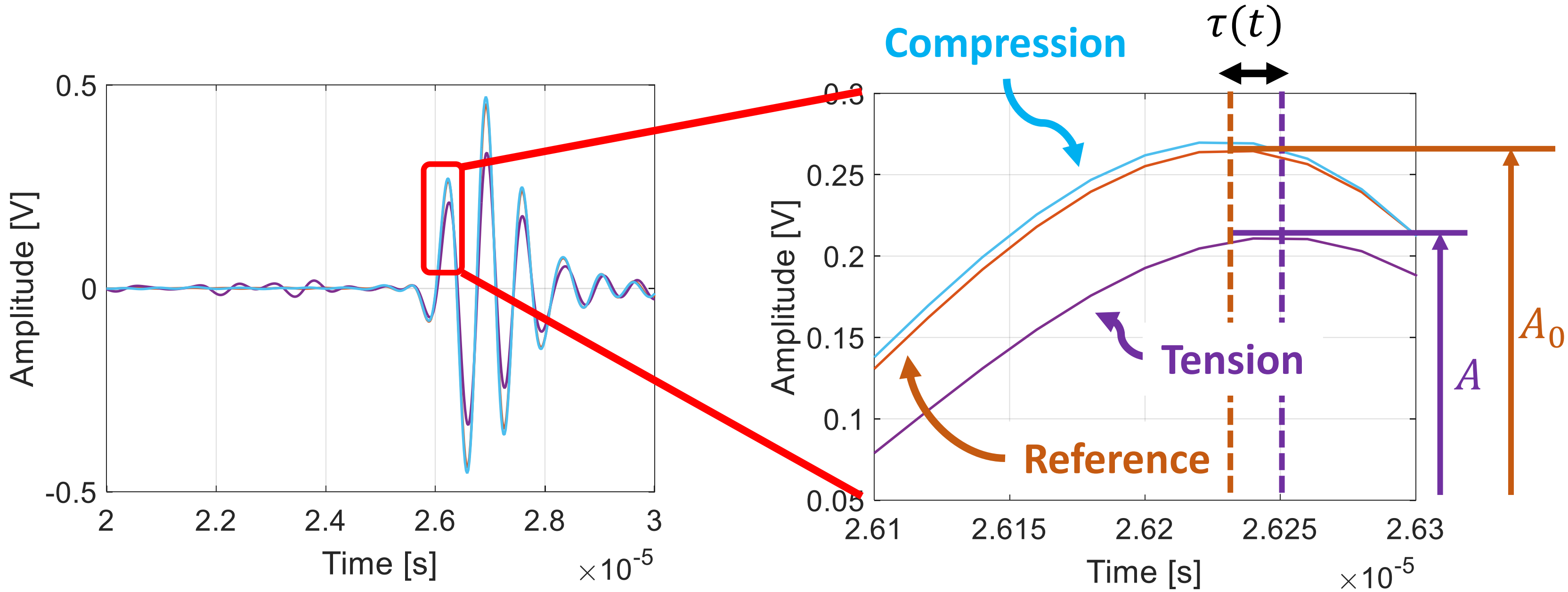


with a thermal pump





DAET: Extract relative variations of pulse velocity and attenuation under dynamic perturbation



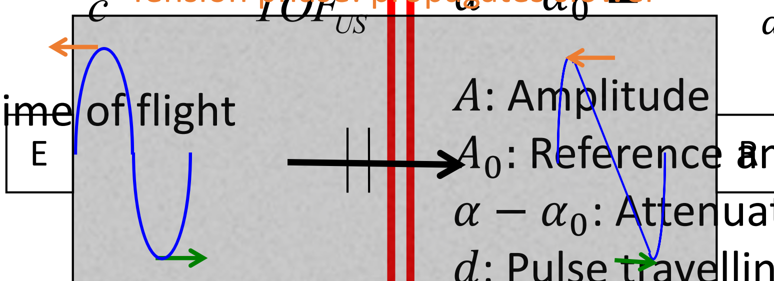
HF pulse velocity change: $\frac{\Delta c}{c}(t) = -\frac{\tau(t)}{TOF_{US}^0}$

$\tau(t)$: Time shift
TOF_{US}^0: Reference of time of flight
Δc: Velocity change
c: Reference velocity

HF pulse relative transmission loss:

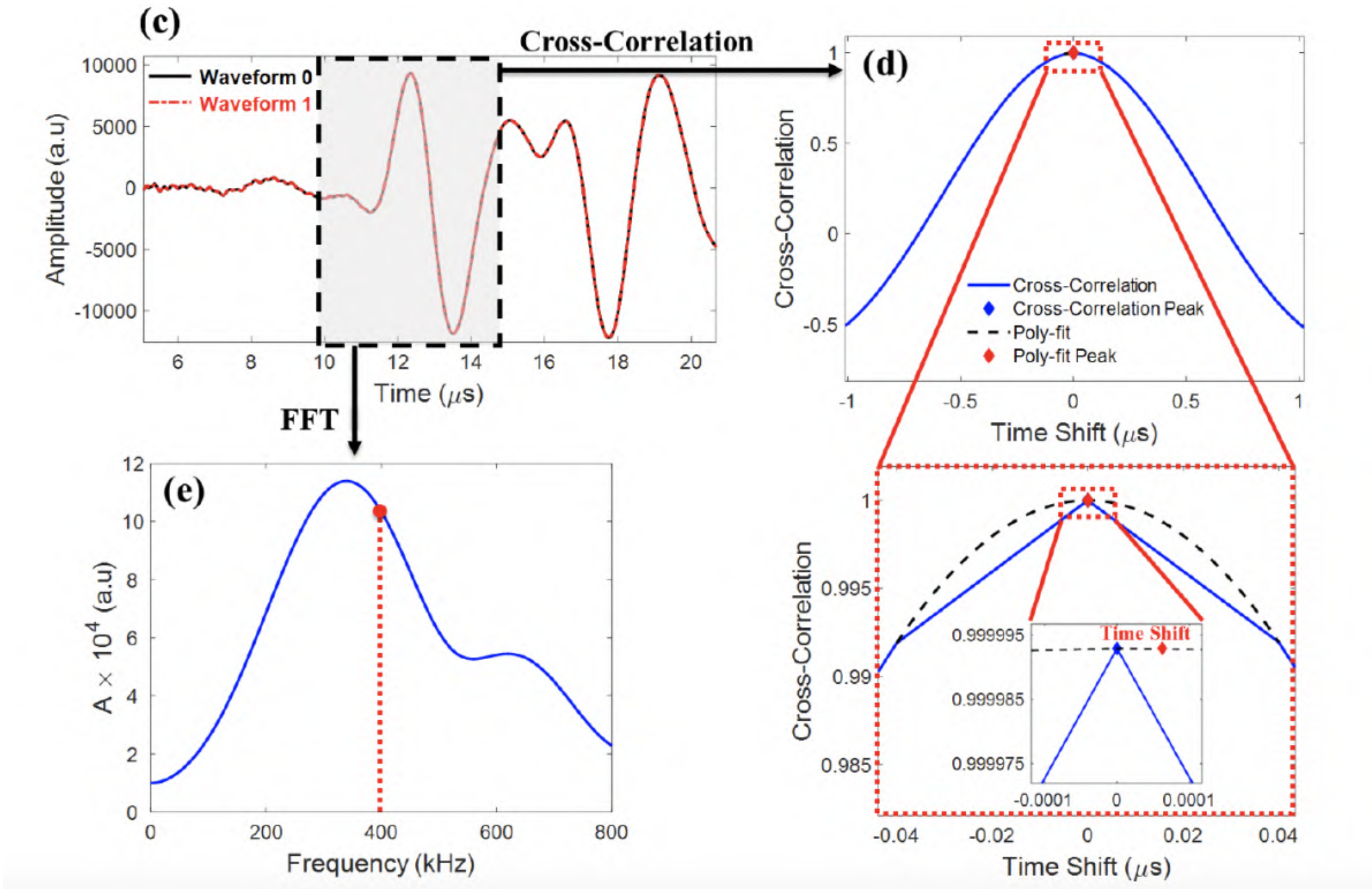
$\alpha - \alpha_0 = -\frac{\ln(A/A_0)}{d}$ [Np/m]

A: Amplitude
A_0: Reference amplitude
$\alpha - \alpha_0$: Attenuation change
d: Pulse travelling distance



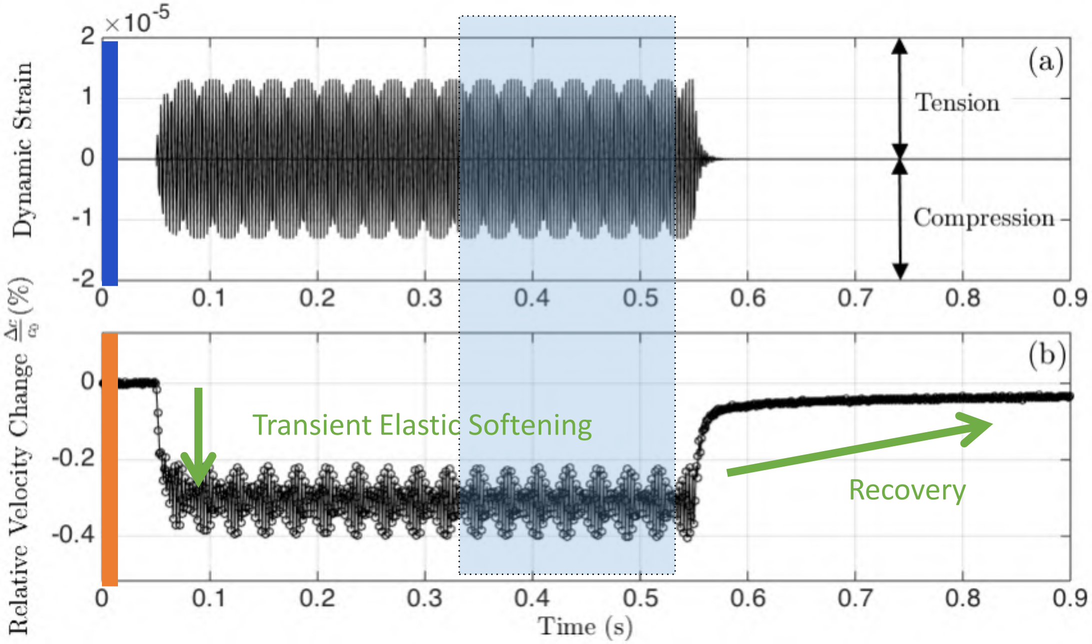
Renaud *et al.*, JASA, 2011 ; Rivière et al, JAP, 2013

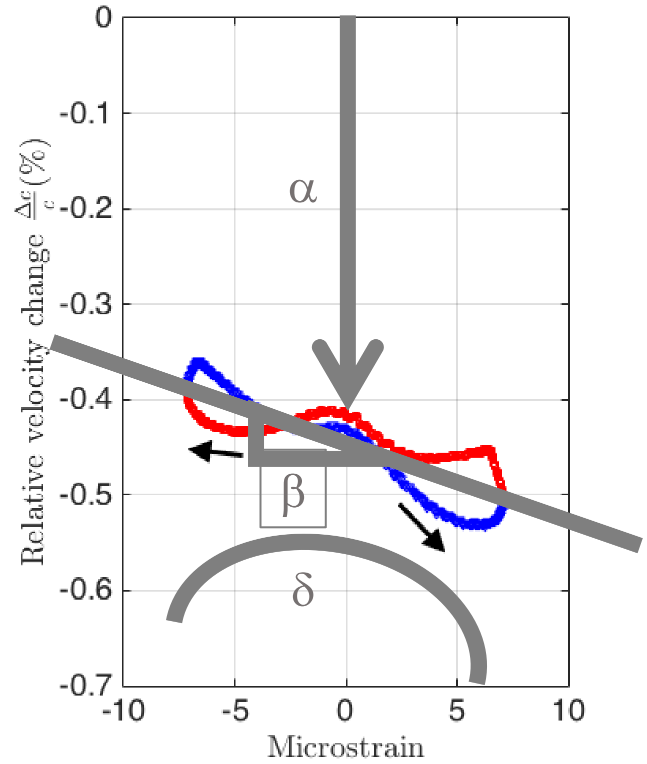
DAET: Extract relative variations of pulse velocity and attenuation under dynamic perturbation



Renaud *et al.*, JASA, 2011 ; Rivière et al, JAP, 2013

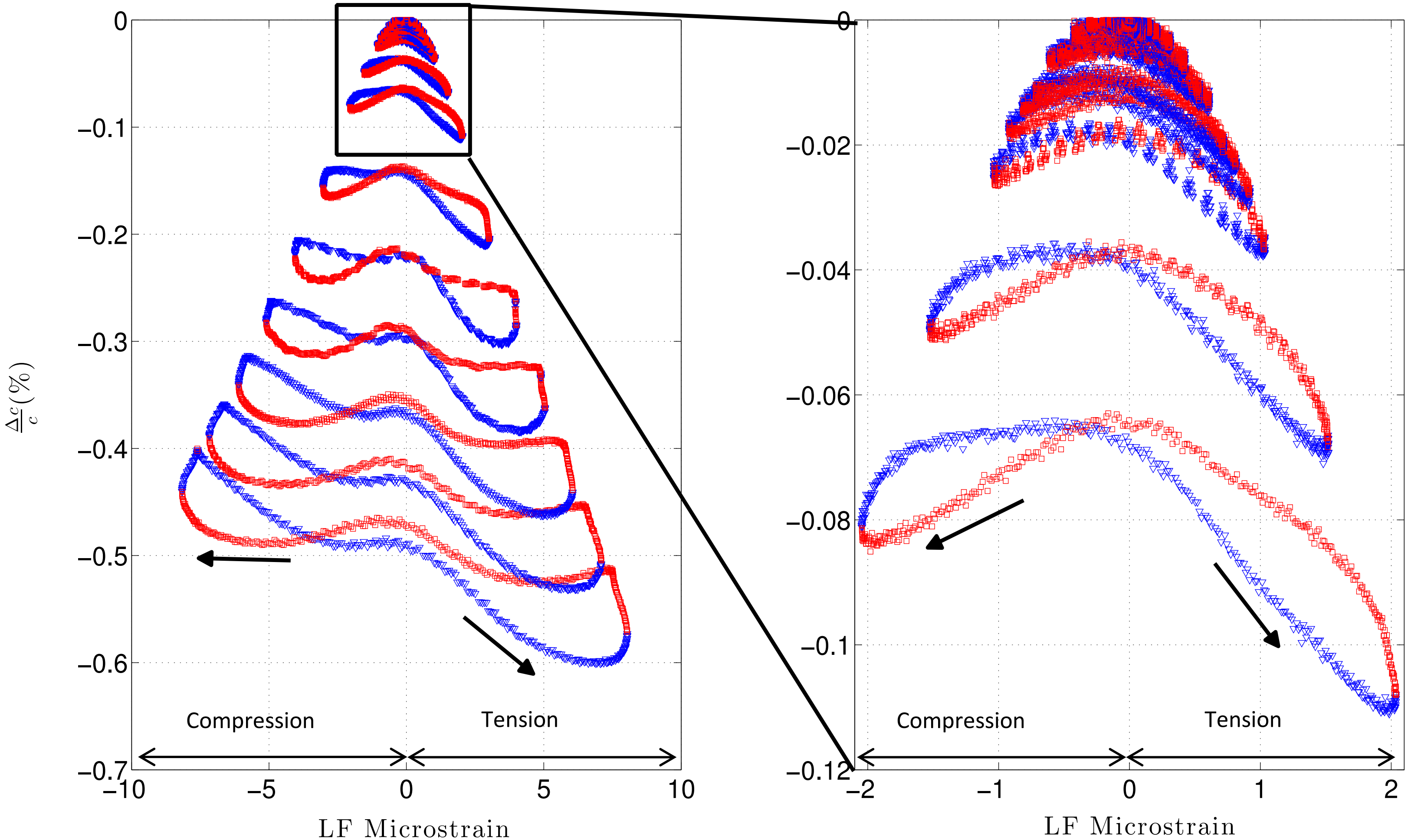
Typical DAET results (in rocks):
Berea sandstone at one pump strain amplitude





Typical DAET results (in rocks):

Berea sandstone at multiple pump strain amplitudes



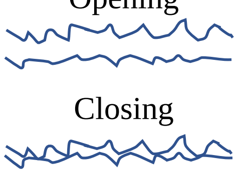
What is the mechanism?

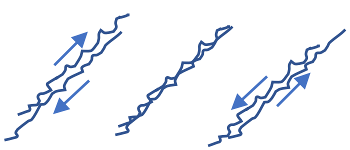
Multiple samples at multiple amplitudes

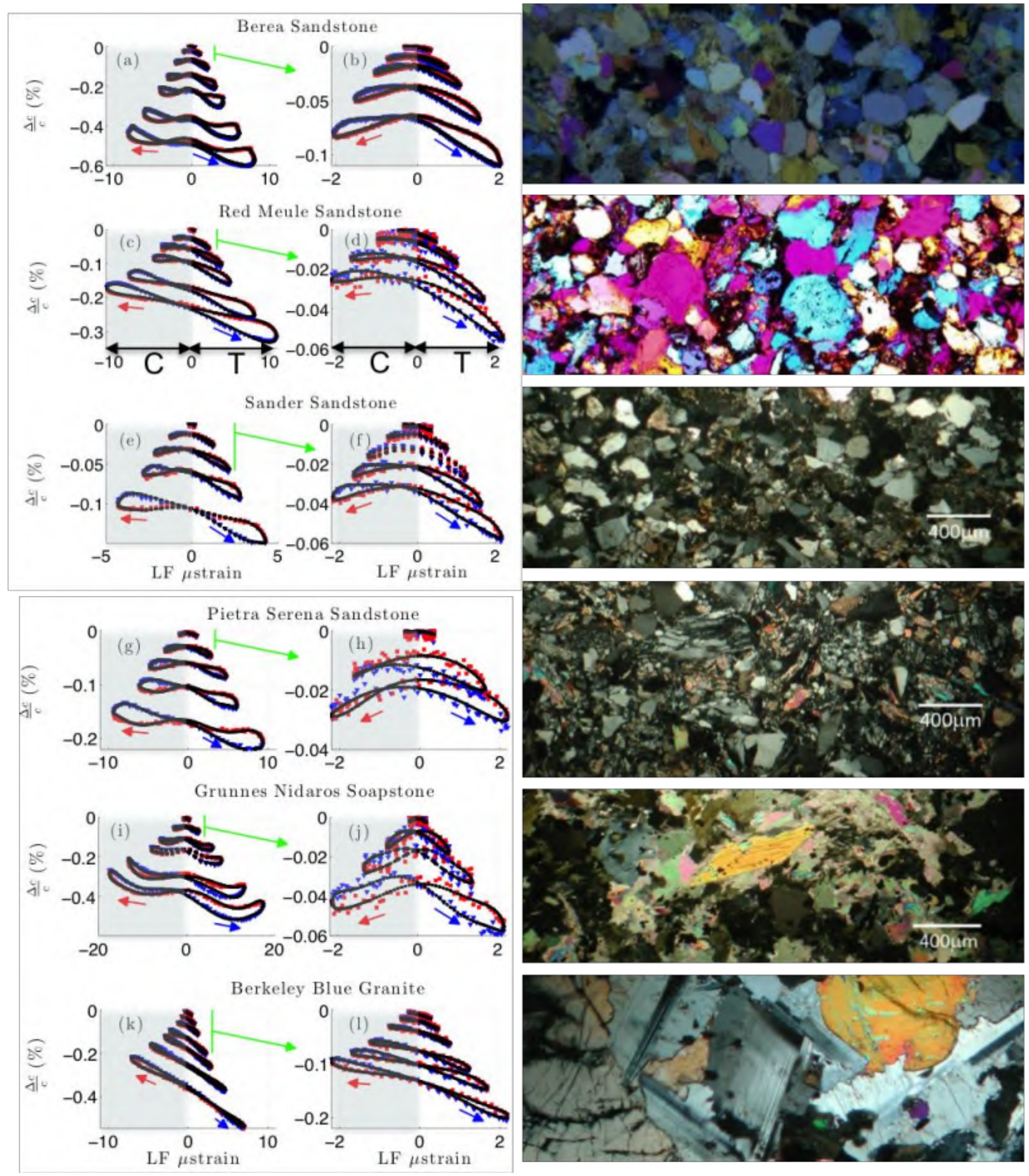
Two clusters suggest two main mechanisms

Slope (β)

All others:
- Average Softening (α)
- Curvature (δ)
- Mexican Hat
- Hysteresis Area







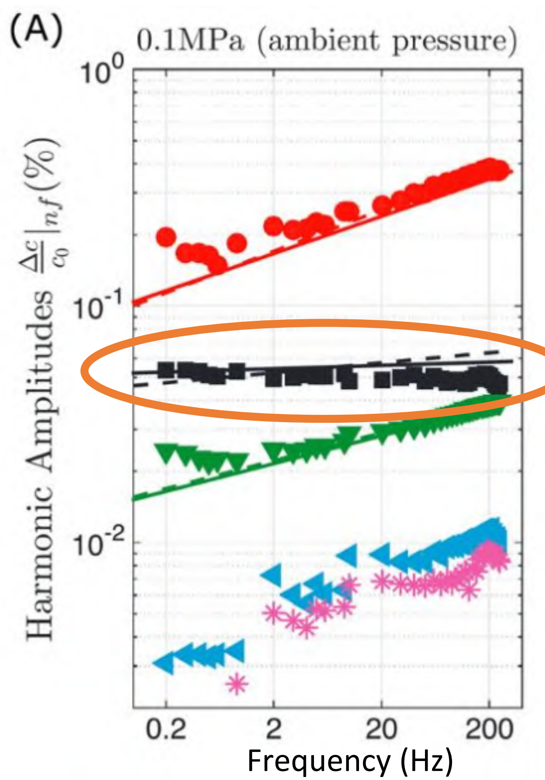
Riviere, Shokouhi, Guyer & Johnson *JGR* (2015)

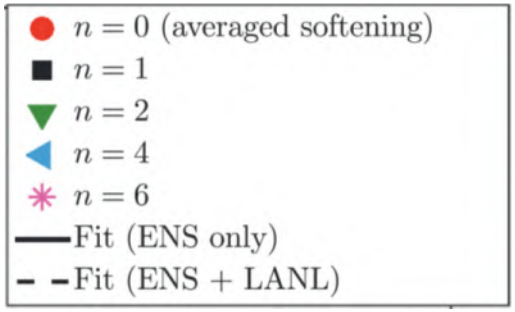
A study of frequency/rate dependence again suggests two mechanism

Again, two main mechanisms

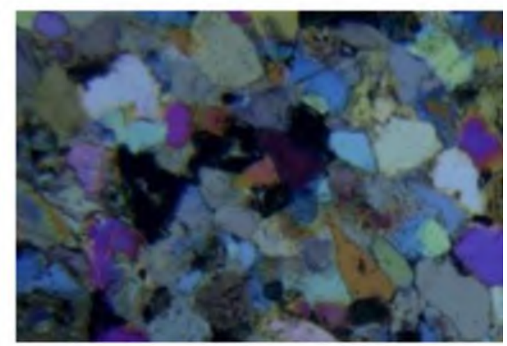
Slope (β)

All others:
- Average Softening (α)
- Curvature (δ)
- Mexican Hat

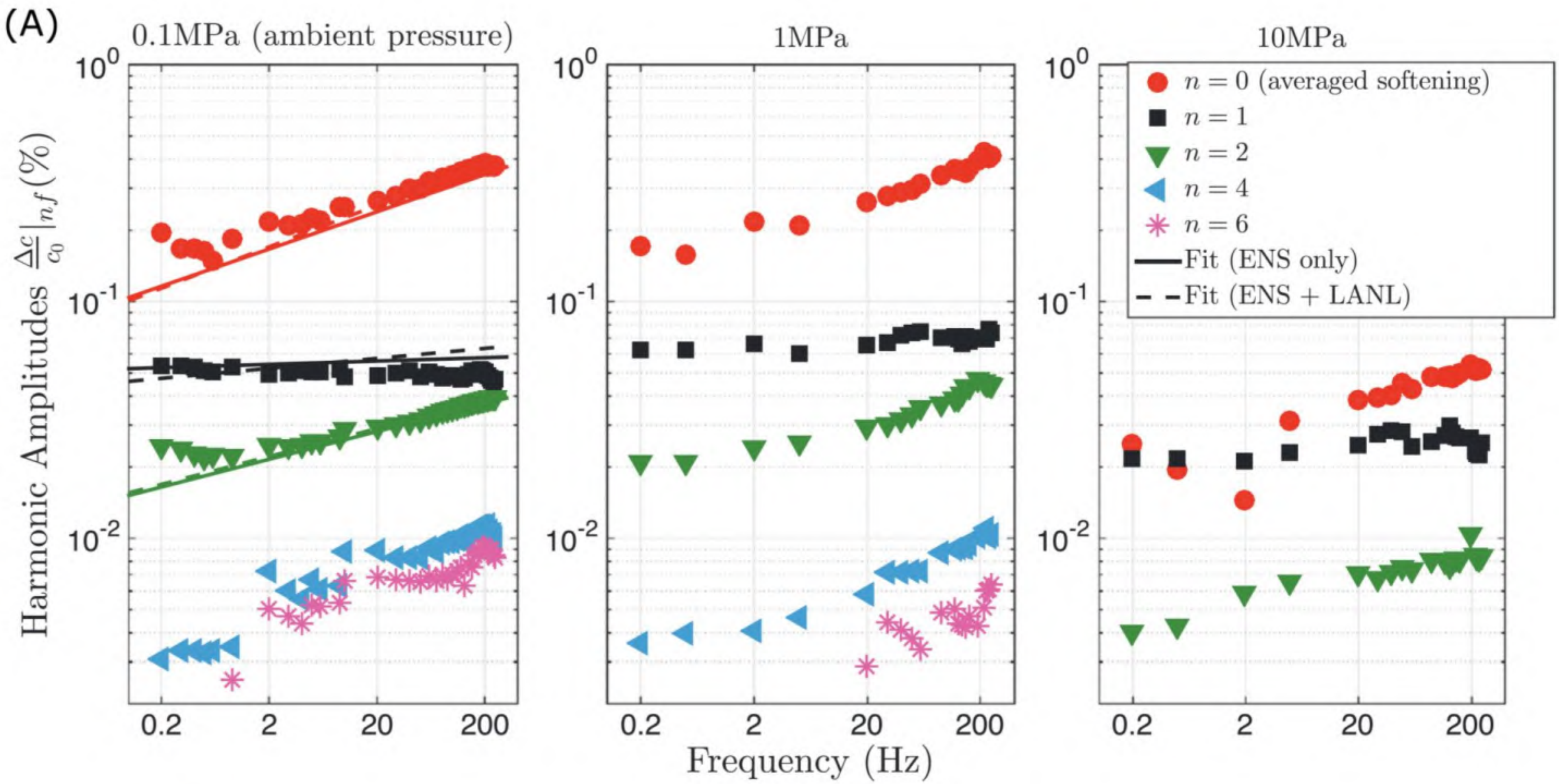




Riviere et al., Geophys. Res. Lett., 2016



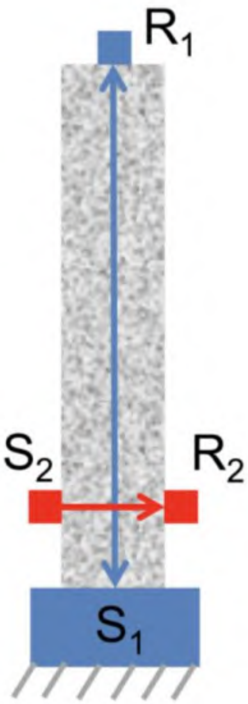
Nonlinearity of intact rock decreases with increasing normal stress

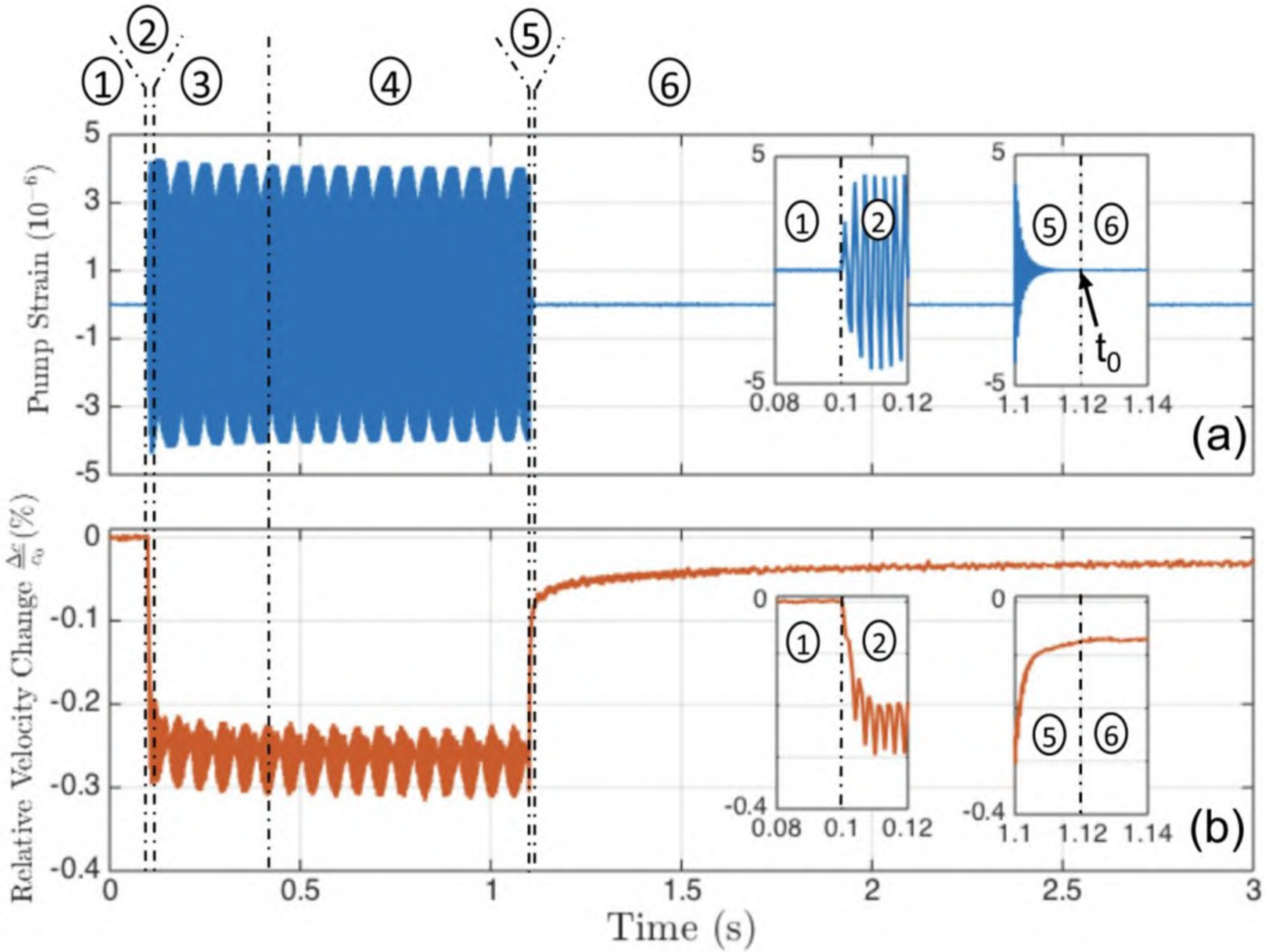


Riviere et al., Geophys. Res. Lett., 2016

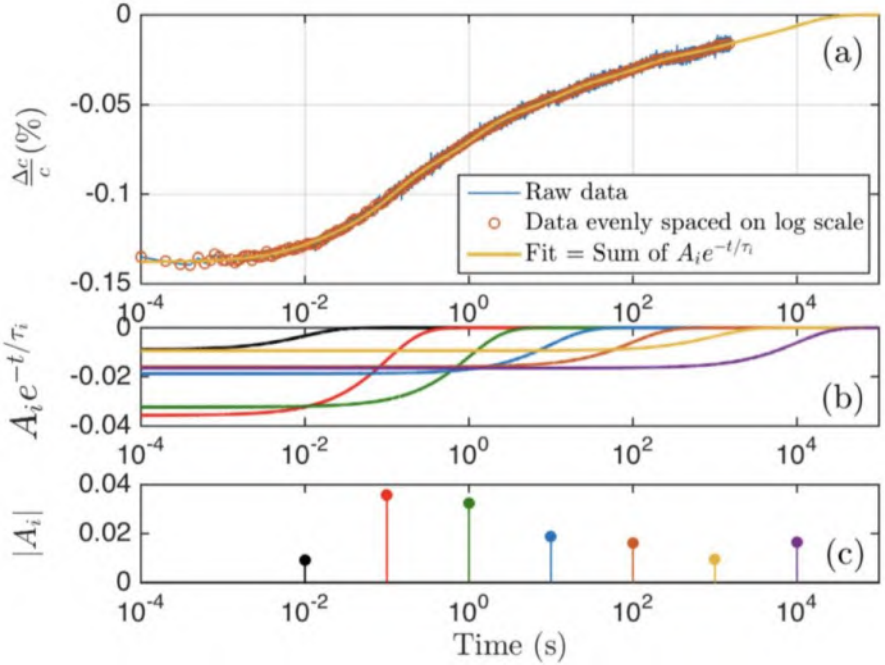
Typical DAET response (in rocks) constitutes 6 regions: focus on slow dynamics (6)

Shokouhi et al., *APL* (2017)

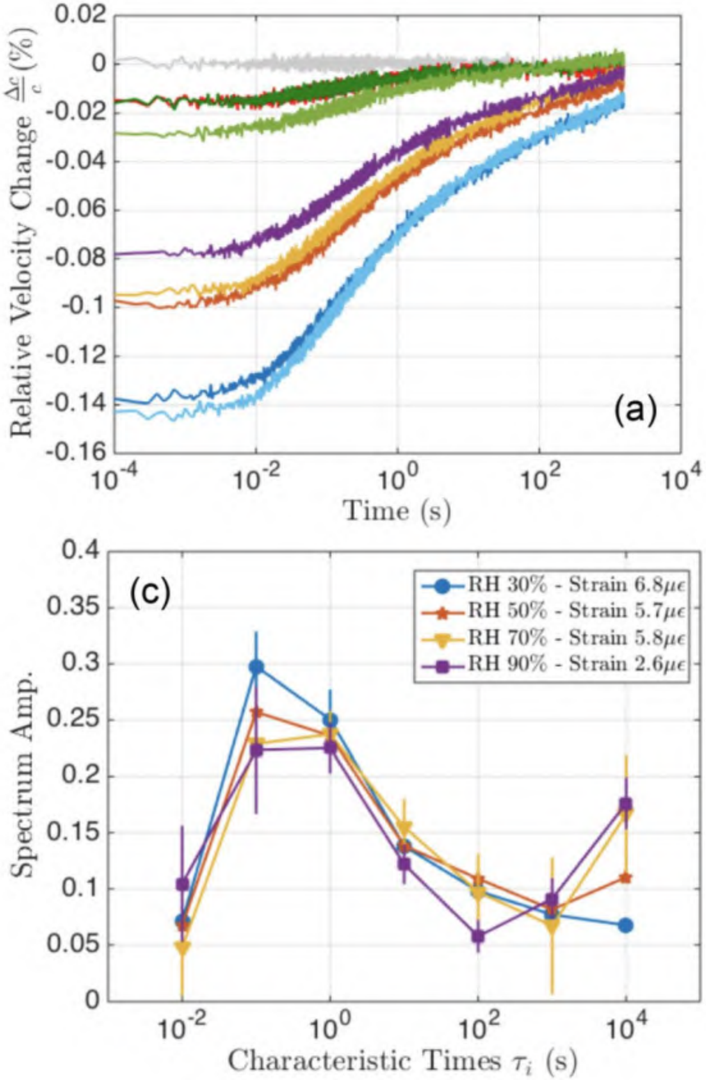




The characteristic time spectrum does not change with RH!



Shokouhi et al., *APL* (2017)



DAET under different stress and saturation conditions

Calibration

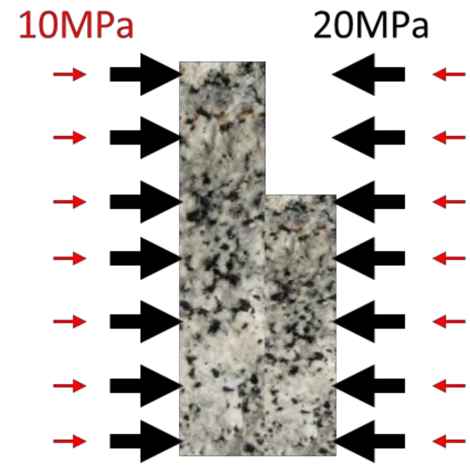
Steel

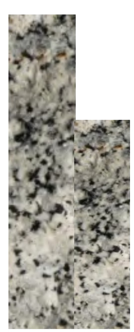
Westerly Granite Sample

Intact

Fractured

Different Stress level



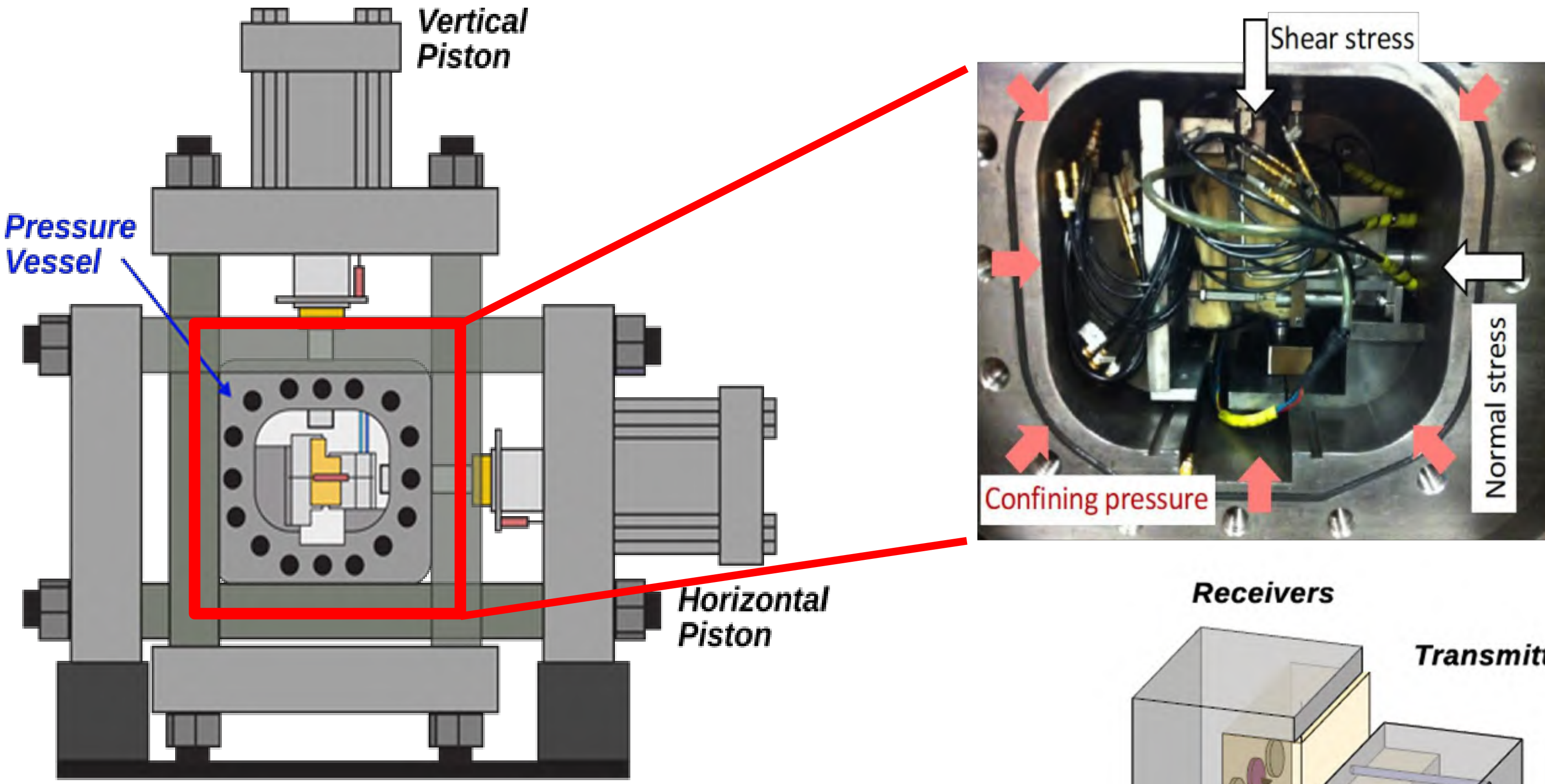


Dry intact

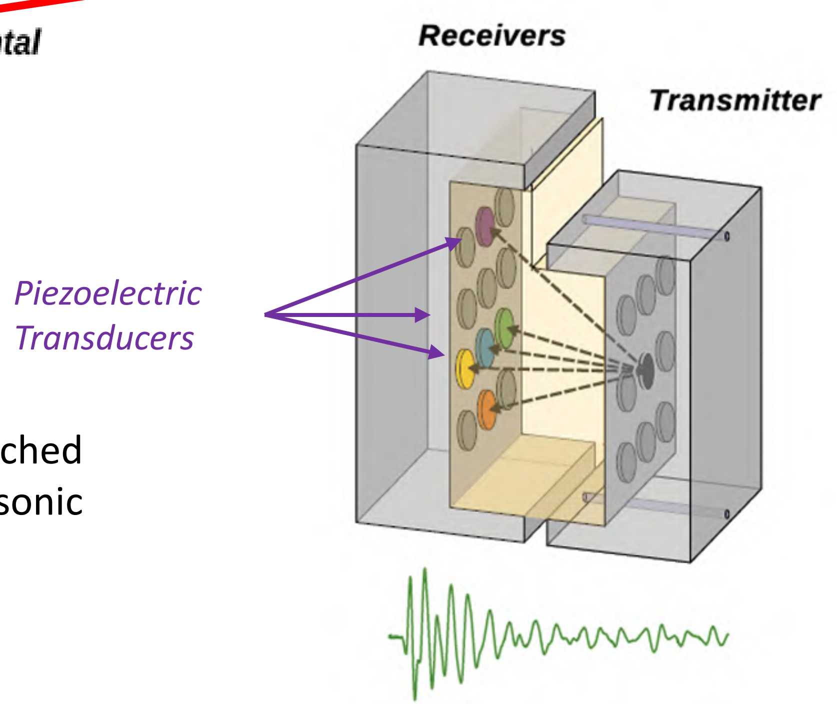
Dry fractured

Saturated Fractured

Experimental Configuration

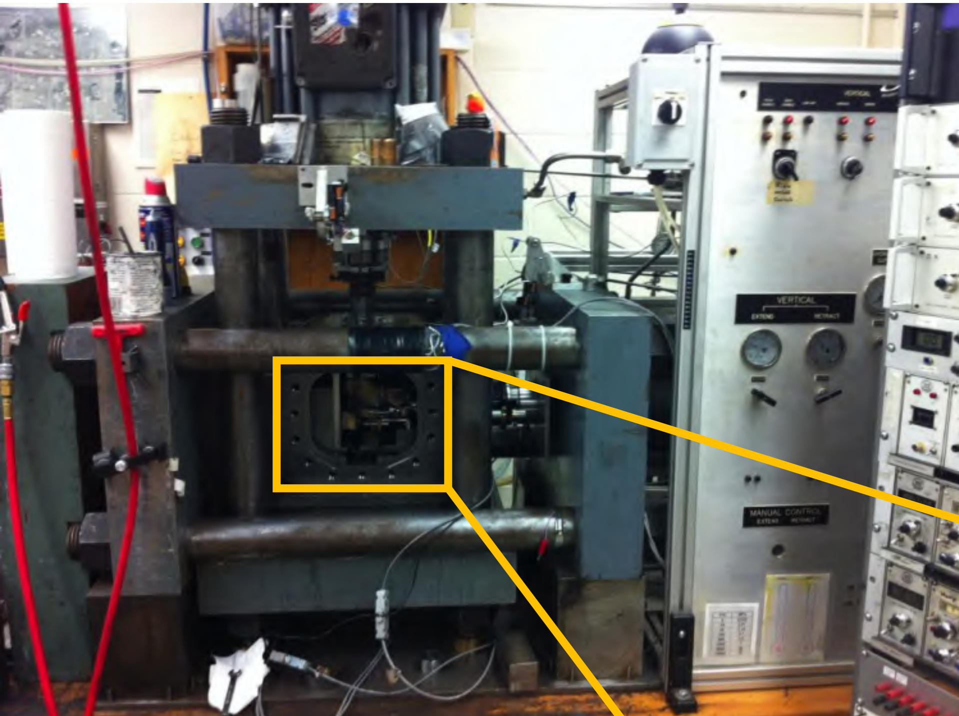


Biaxial Deformation Apparatus

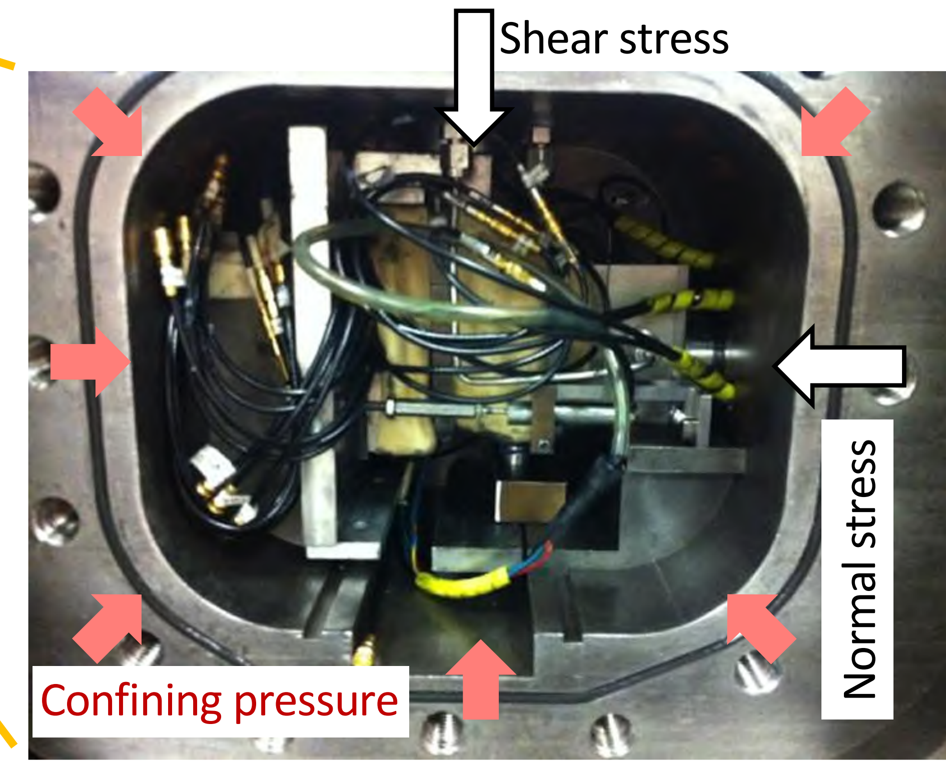


L-shaped Westerly Granite Sample, sandwiched between steel platens with embedded ultrasonic transducers.

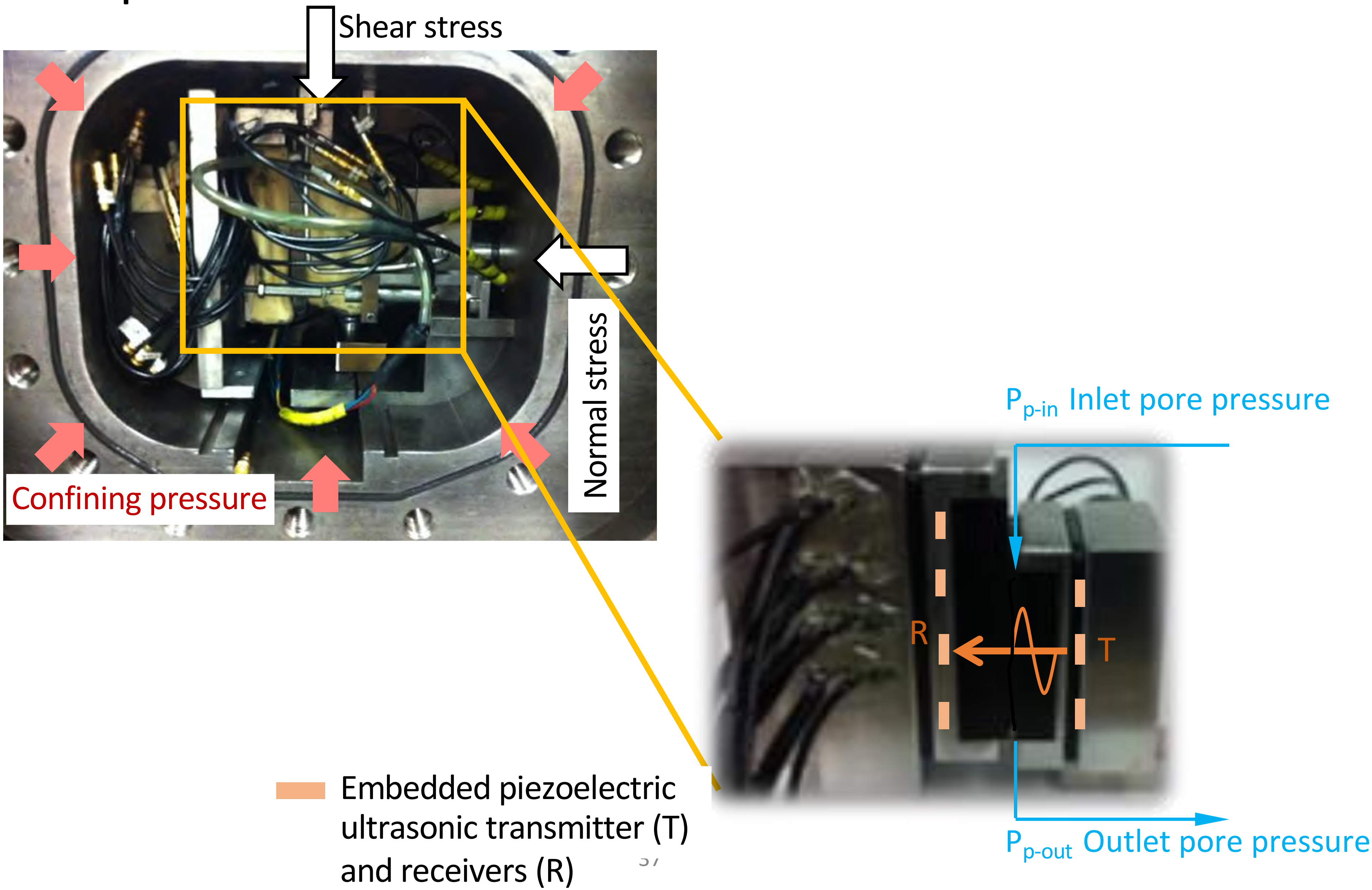
We study L-shape samples in a triaxial cell and a single-direct shear configuration



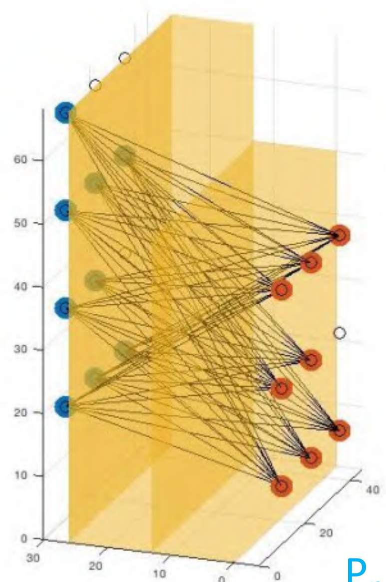
Biaxial loading apparatus



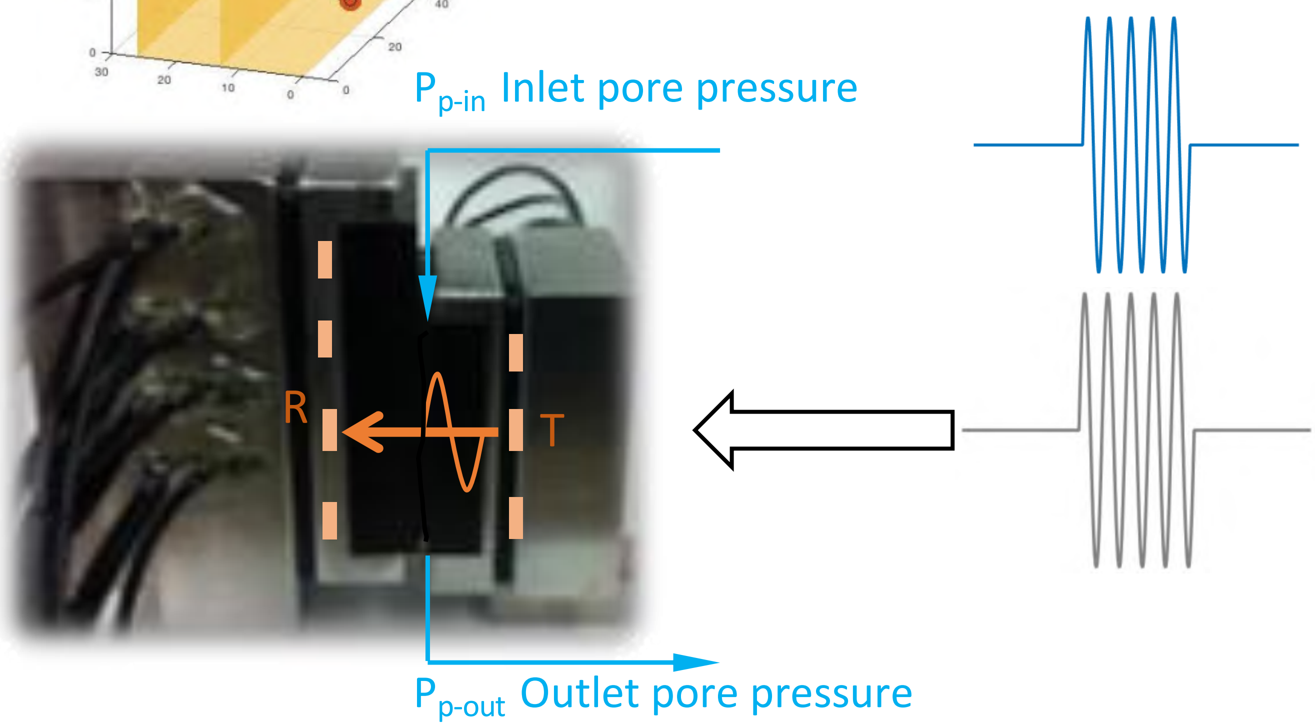
L-shaped Westerly Granite Sample is sandwiched between steel platens with embedded ultrasonic transducers.



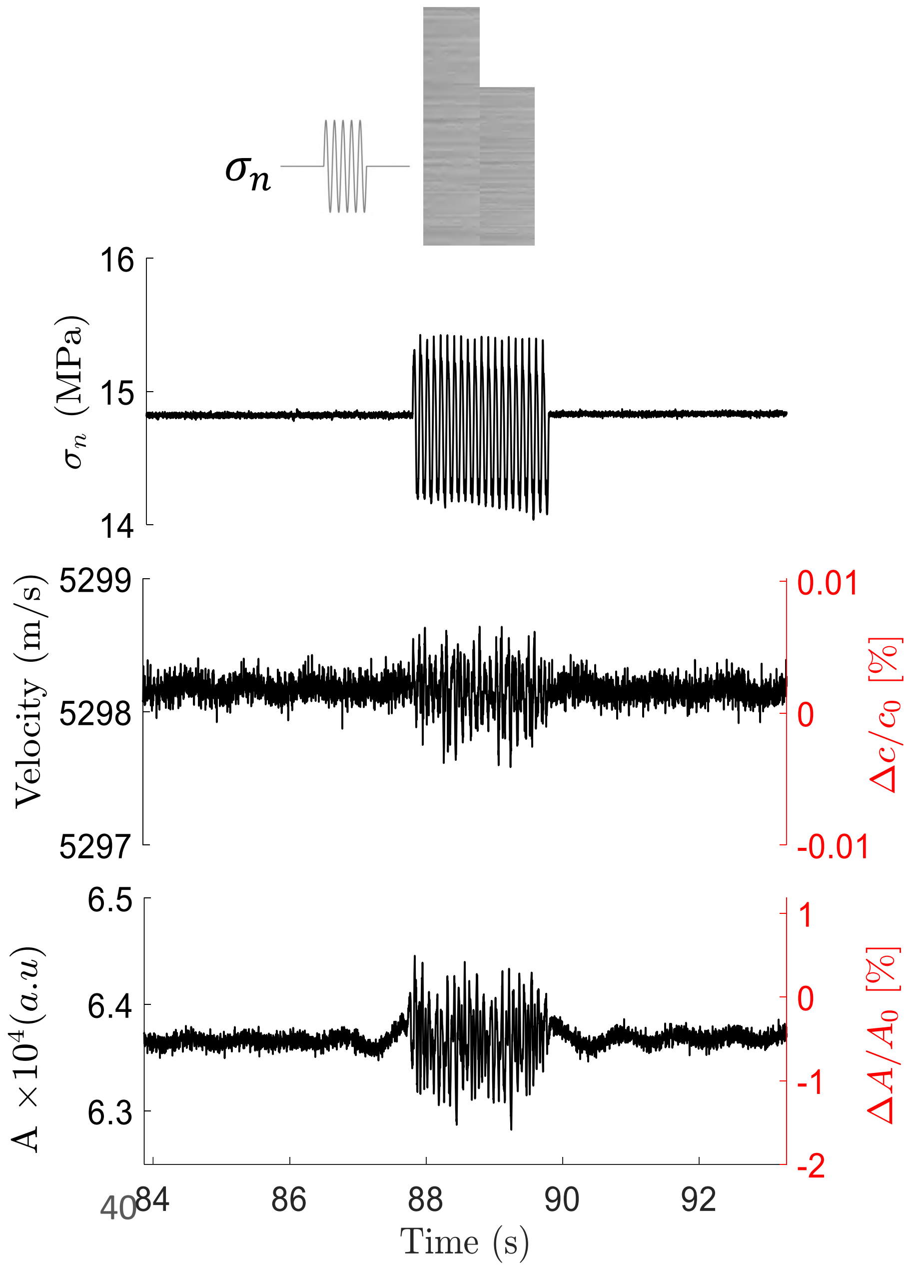
An ultrasonic array is used to monitor the stiffness of the fractured rock under 'effective' stress oscillations



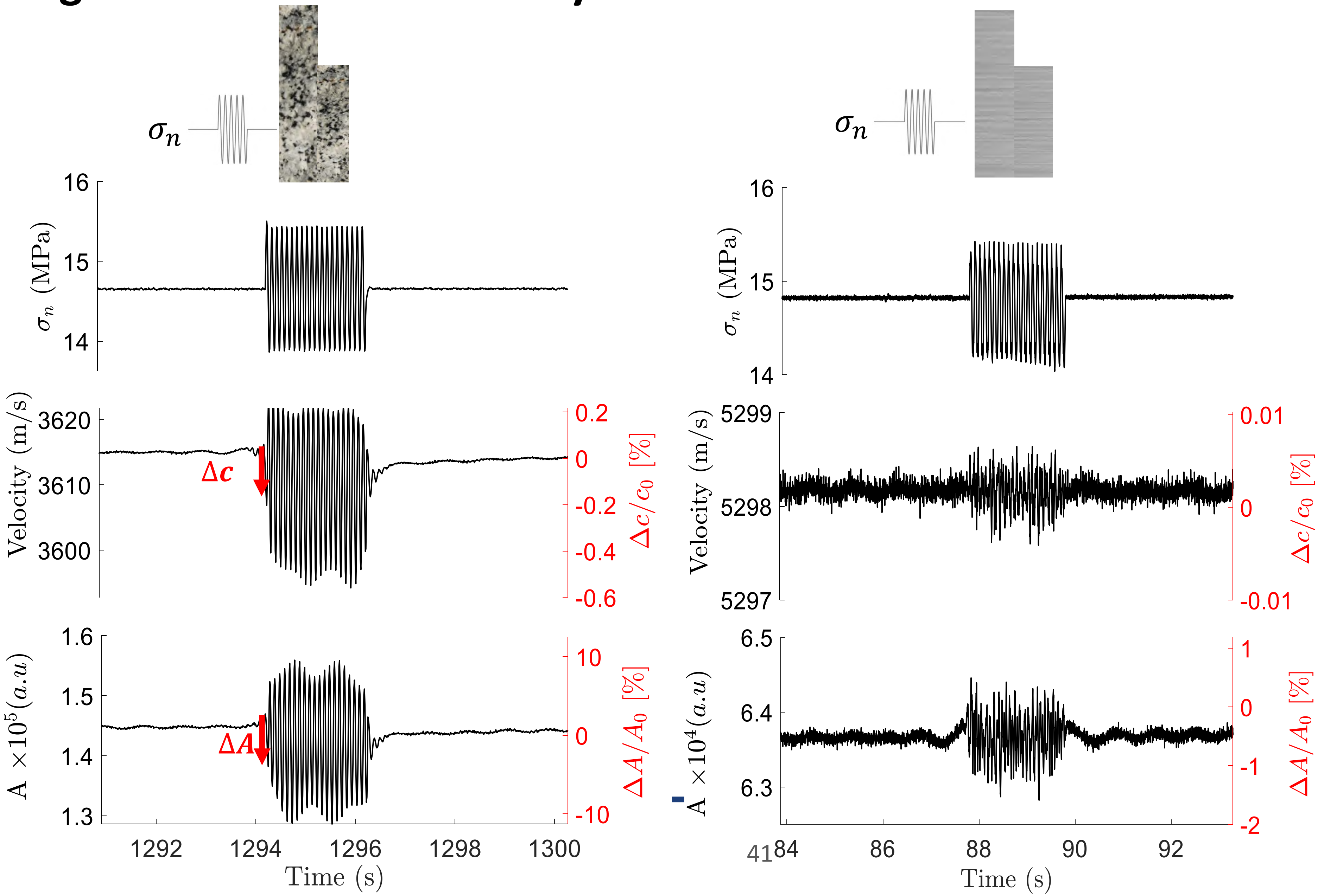
$$s_{eff} = s_n - P_p$$



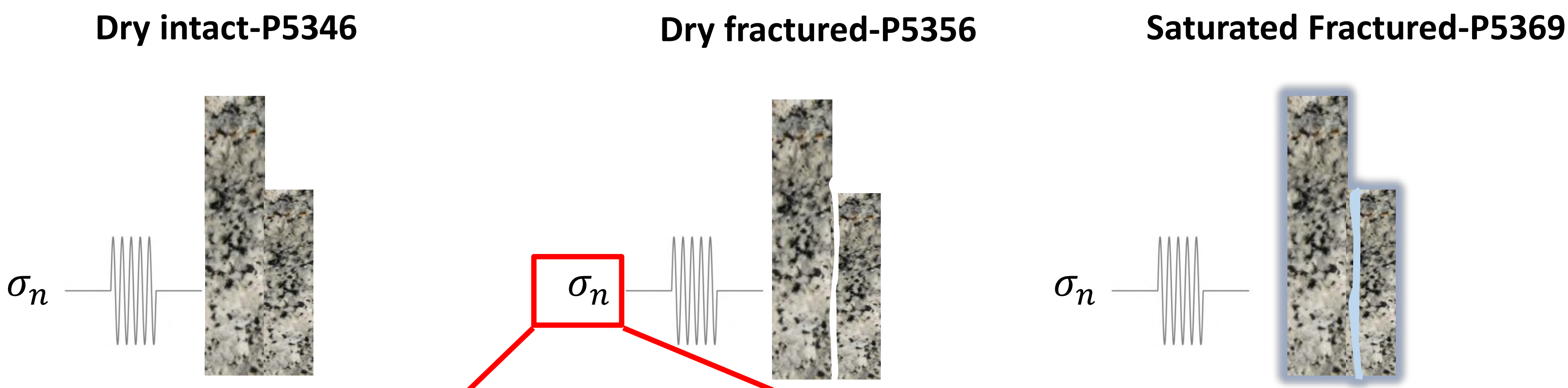
Calibration run: Westerly granite exhibits one order of magnitude more nonlinearity than steel !

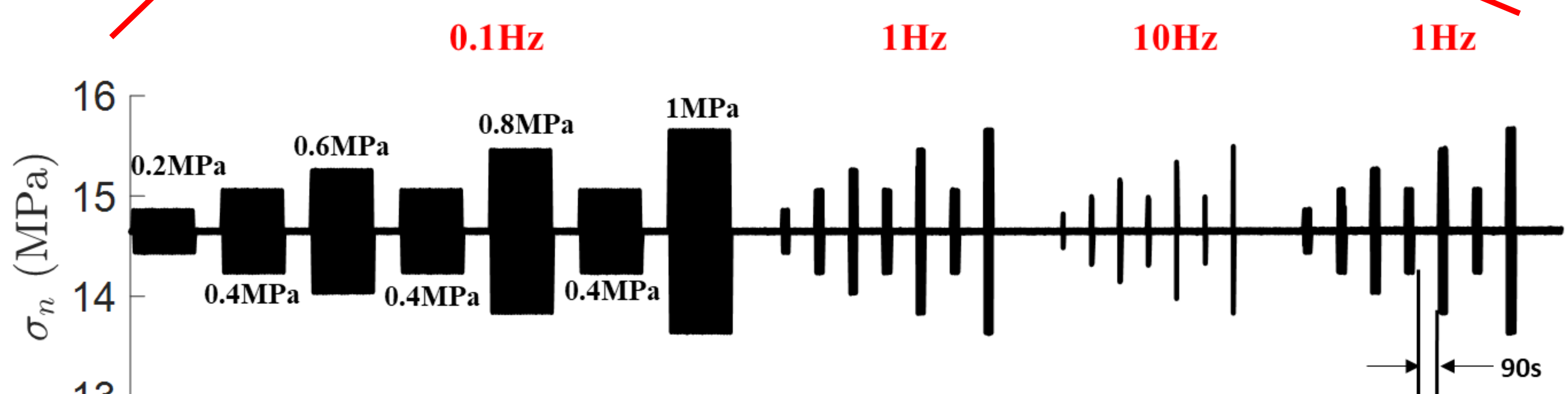


Calibration run: Westerly granite exhibits one order of magnitude more nonlinearity than steel !



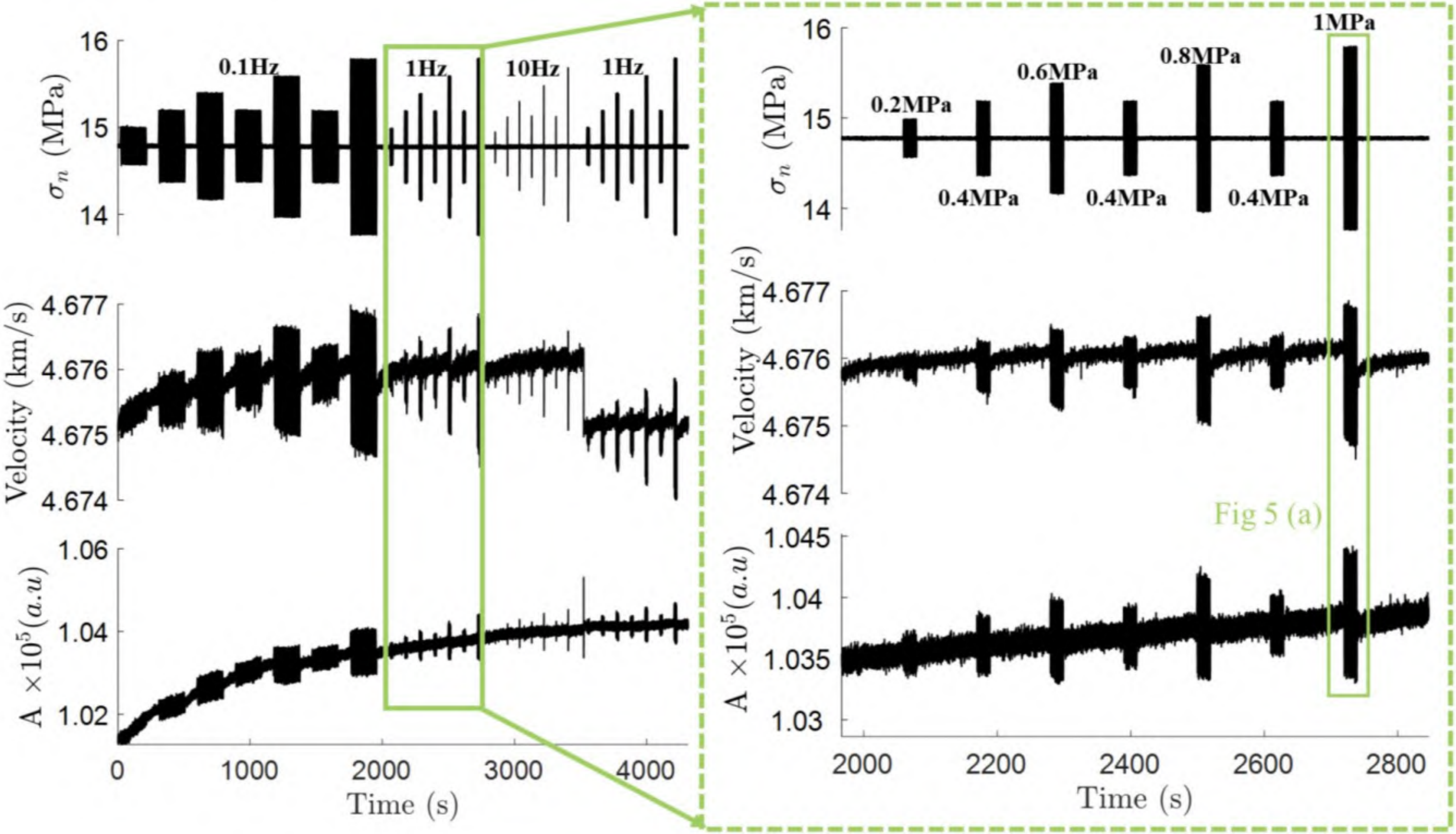
Experiment Protocol Overview





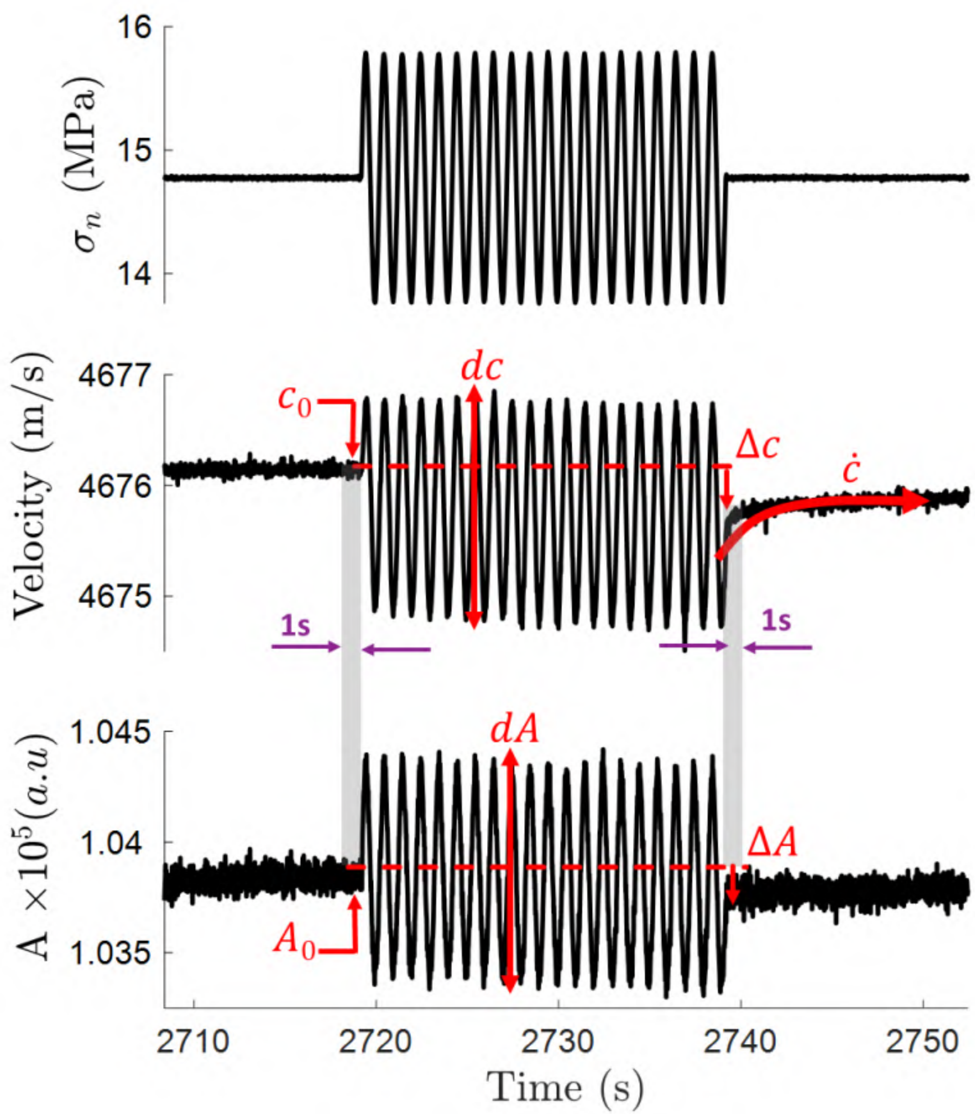
Overview of imposed normal stress oscillations on the sample.

Manogharan et al., *JMPS* 2021



0.1Hz
1Hz
10Hz
1Hz
16
15
14
σn (MPa)
4.677
4.676
4.675
4.674
Velocity (km/s)
1.06
1.04
1.02
A ×10^5(a.u)
0
1000
2000
3000
4000
Time (s)
0.2MPa
0.4MPa
0.6MPa
0.4MPa
0.8MPa
0.4MPa
1MPa
1.045
1.04
1.035
1.03
2000
2200
2400
2600
2800
Fig 5 (a)

We measure nonlinear elasticity in terms of ...

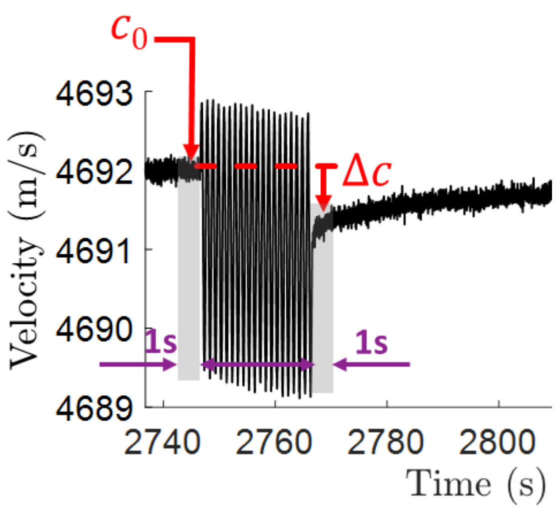


(1) $\Delta c / c_0$ - offset

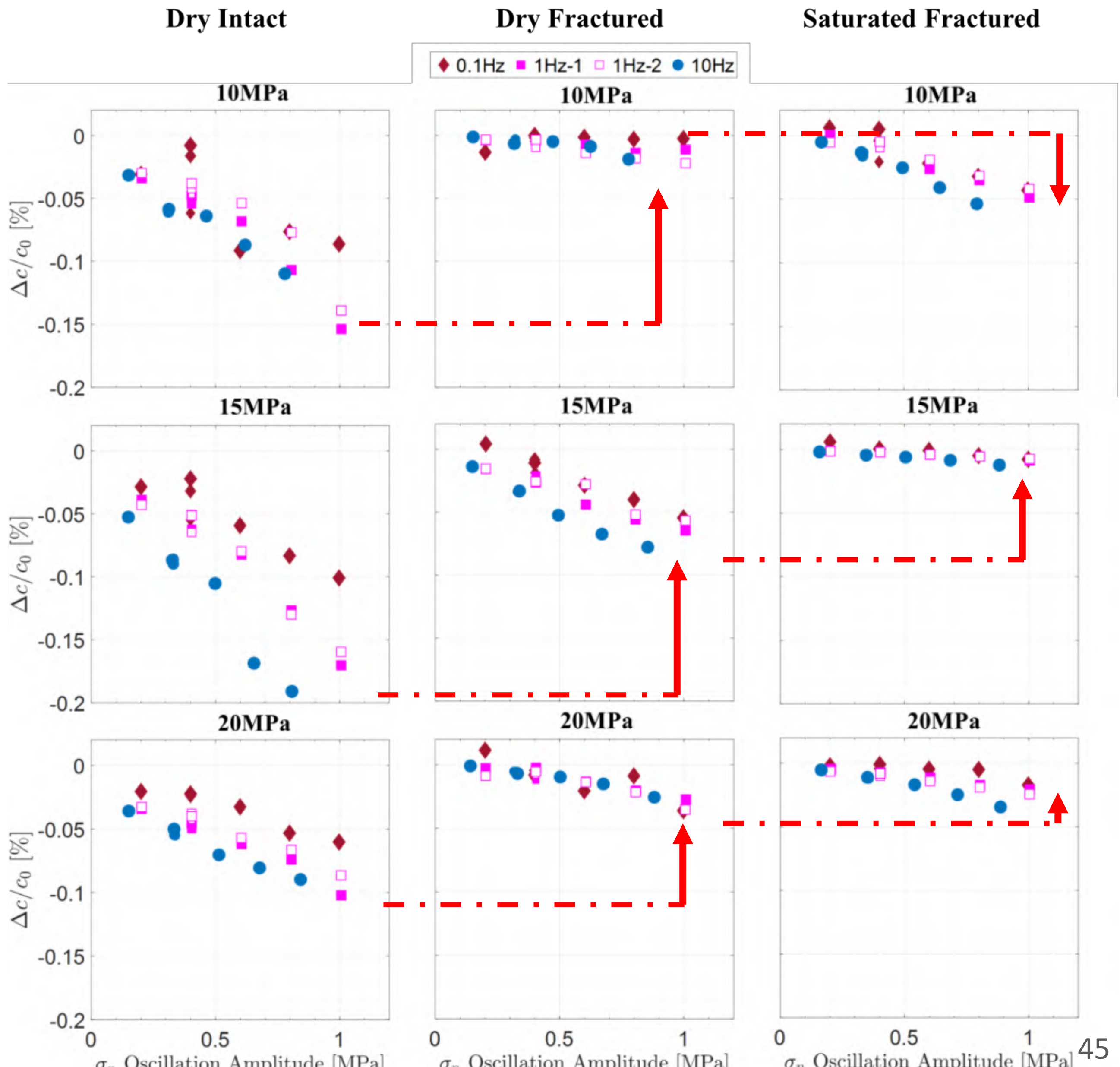
(2) dc / c_0- Amplitude change

(3) $\dot{c}$ Recovery rate

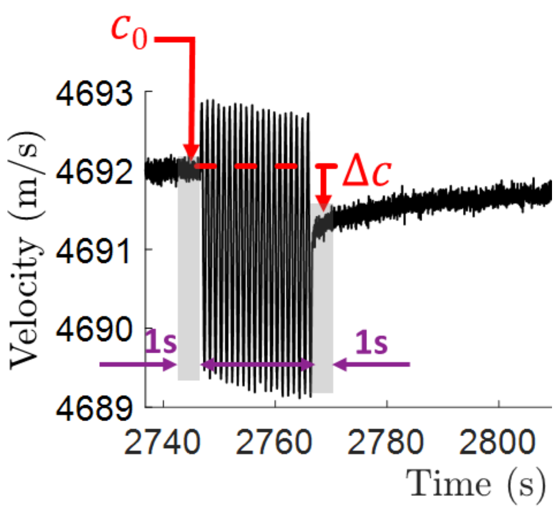
Relative velocity change ($\Delta c / c_0$) is **largest** for intact sample!



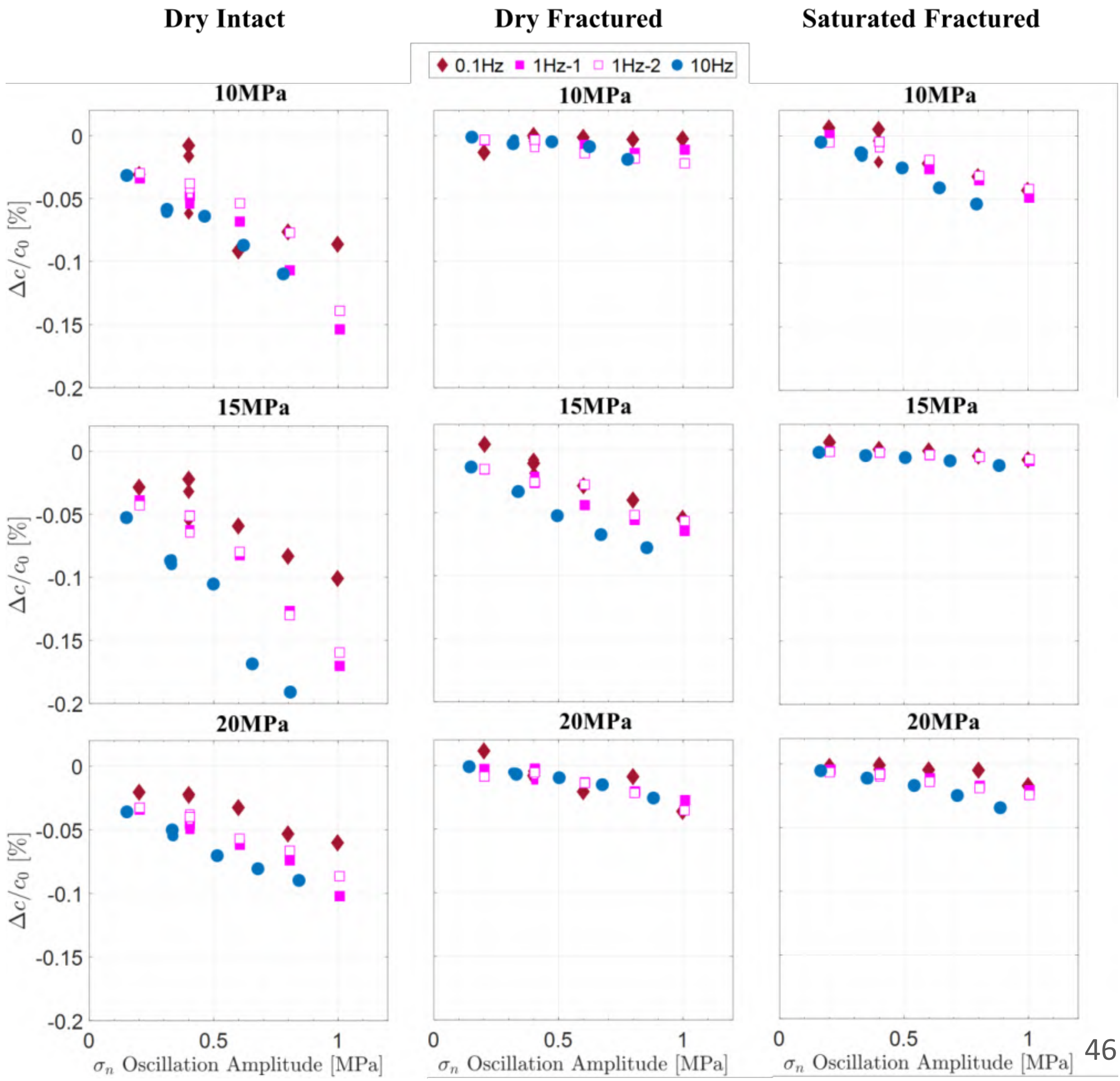
Manogharan et al., *JMPS* 2021



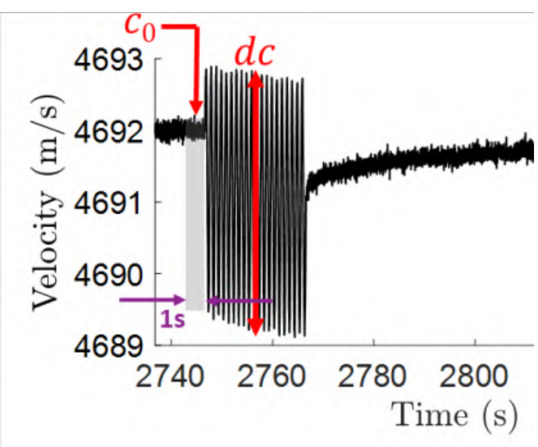
Relative velocity change ($\Delta c / c_0$) is largely frequency dependent!



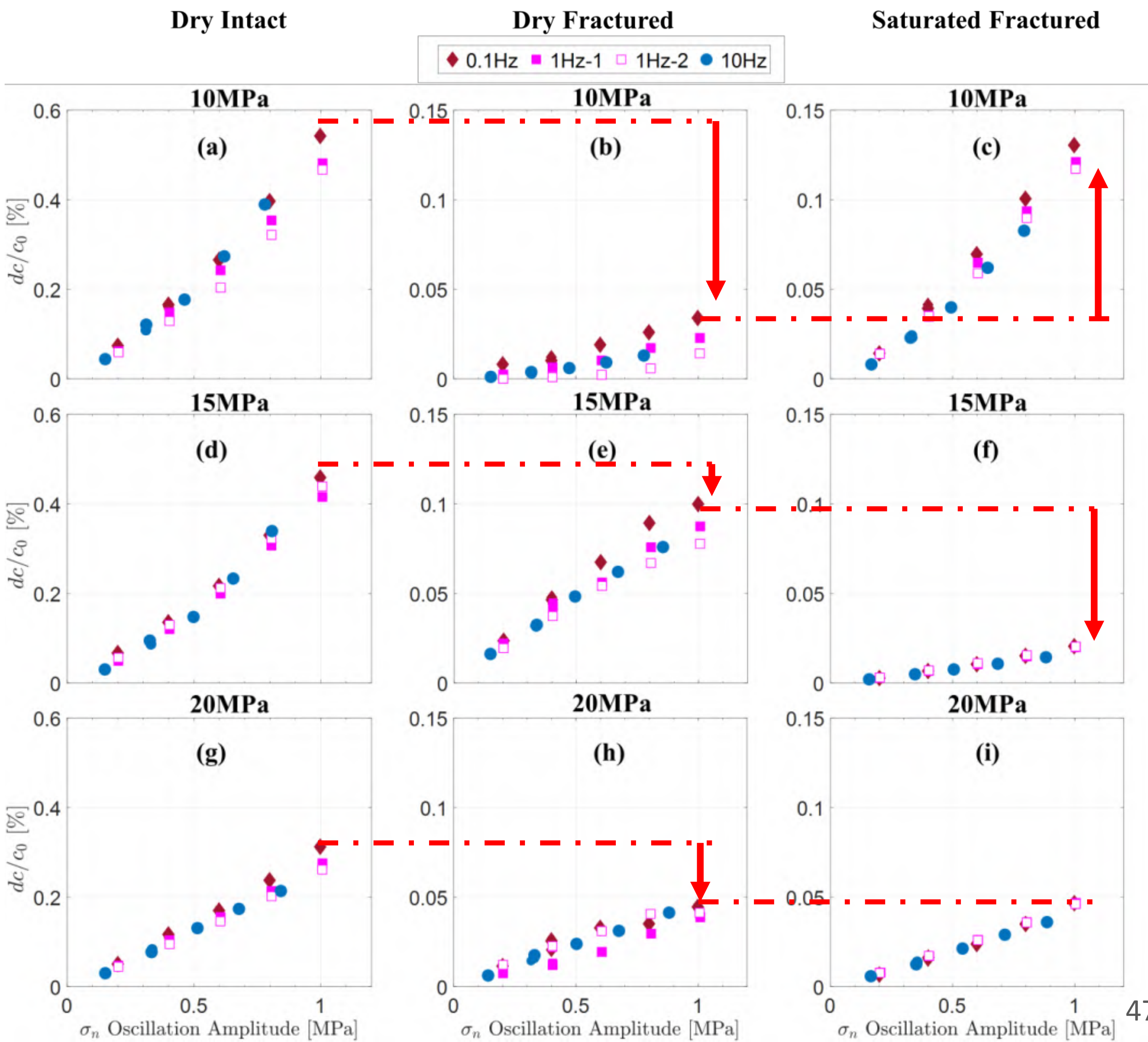
Manogharan et al., *JMPS* 2021



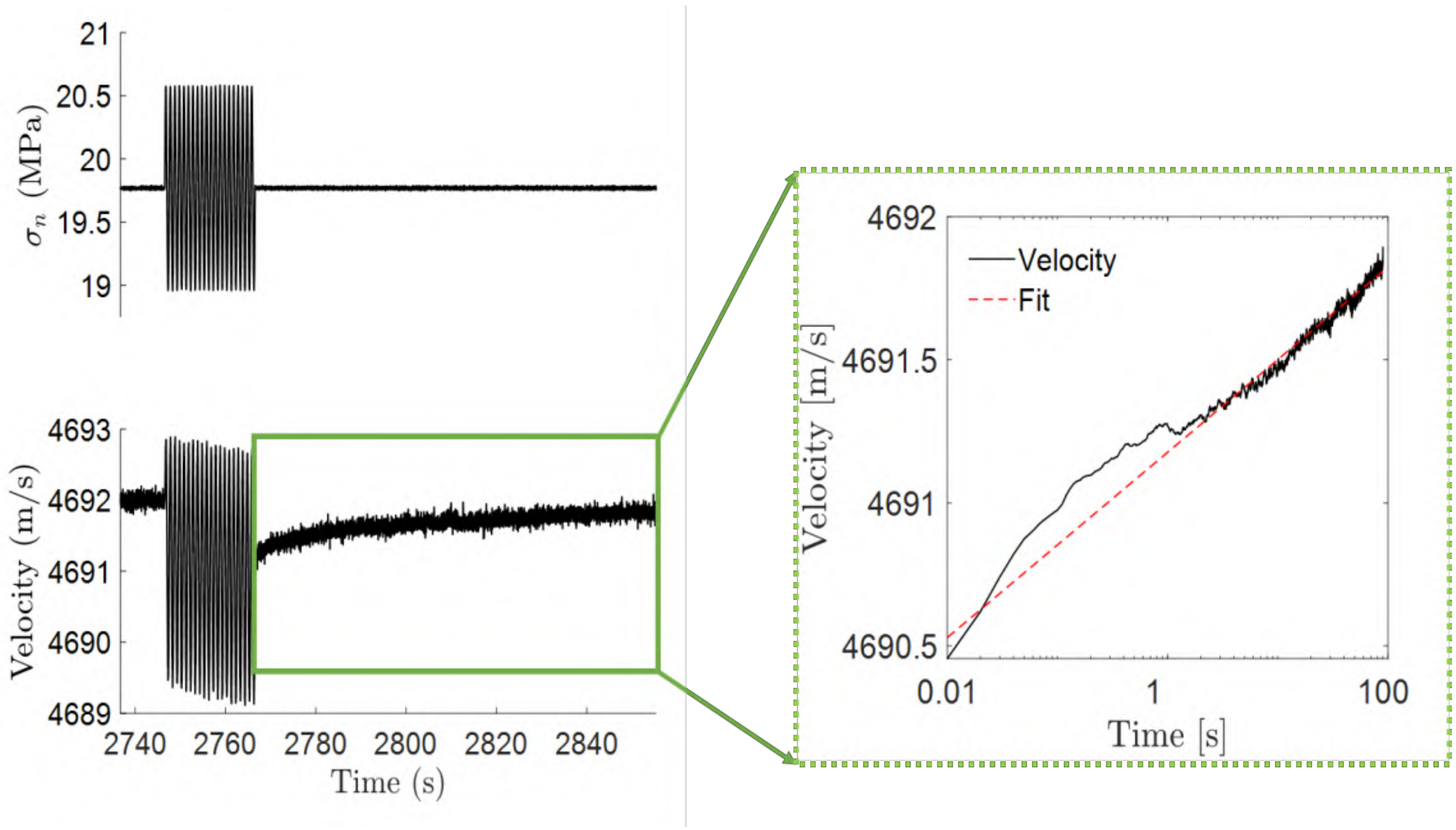
Wave velocity modulation amplitude (dc/c_0) shows similar trends to $(\Delta c/c_0)$ but less frequency dependency and scatter



Manogharan et al., *JMPS* 2021



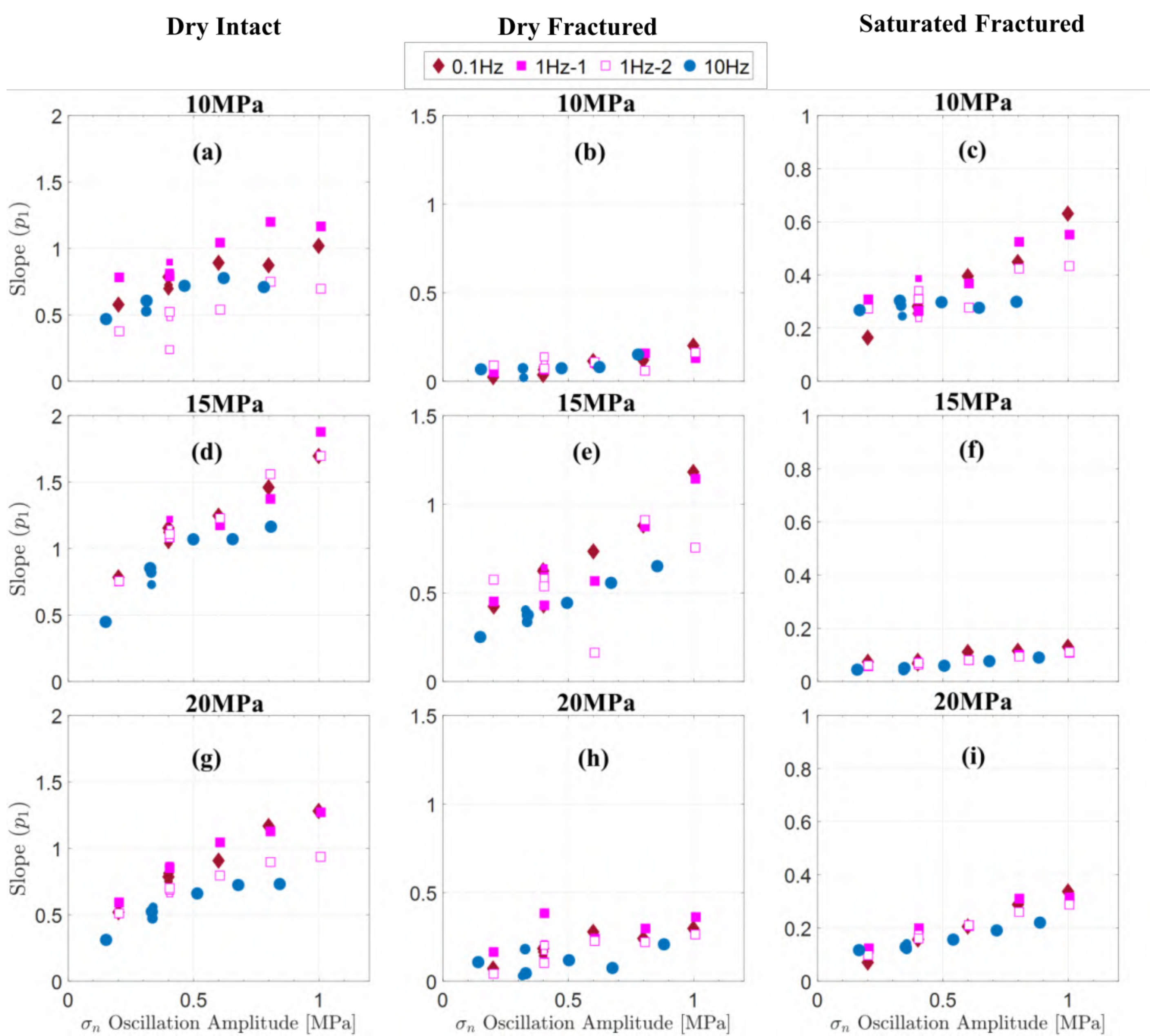
Recovery of wave velocity follows a time logarithmic relaxation



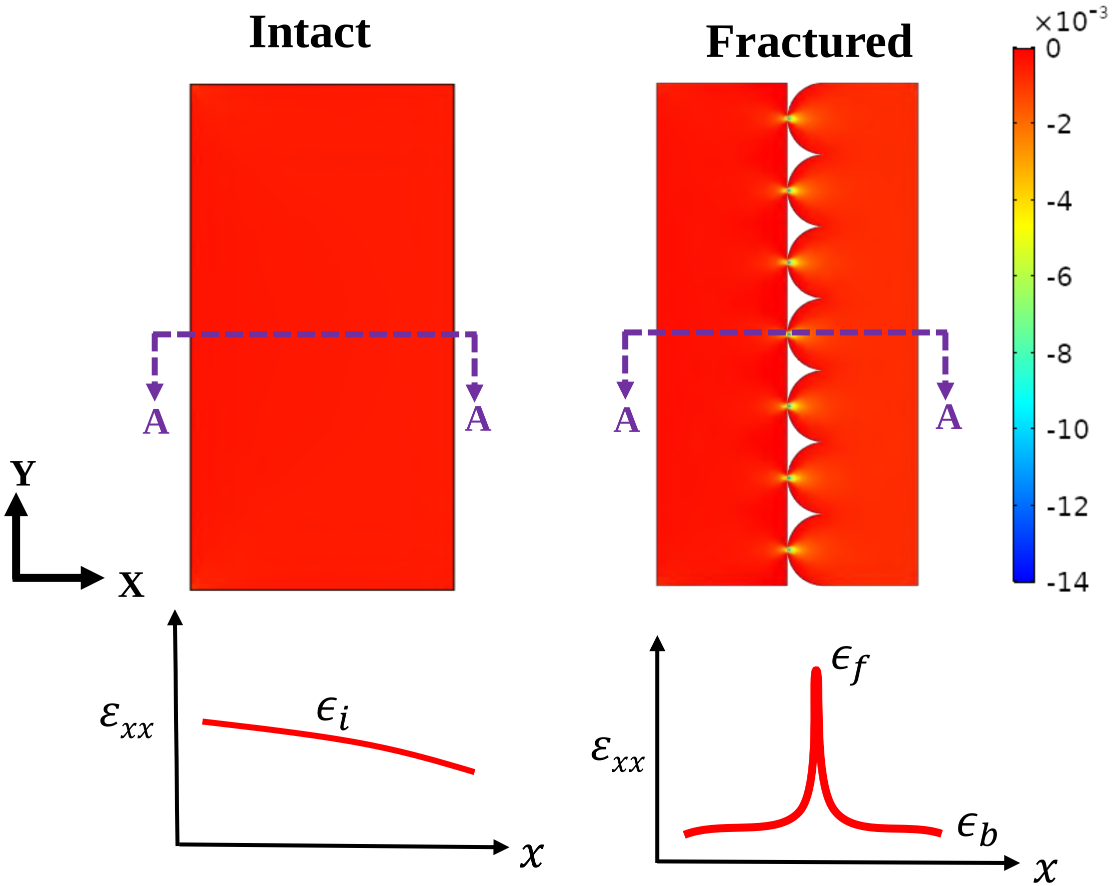
Manogharan et al., *JMPS* 2021

Red line is the linear fit $c = p_1 \log(t) + p_2$, to late-time recovery. where p_1 and p_2 are the slope (recovery rate) and intercept.

Recovery rate of wave velocity ($slope\ p_1$) is generally in accord with $\Delta c/c_0$ and dc/c_0



Why dry fractured sample exhibit less nonlinearity than intact sample?

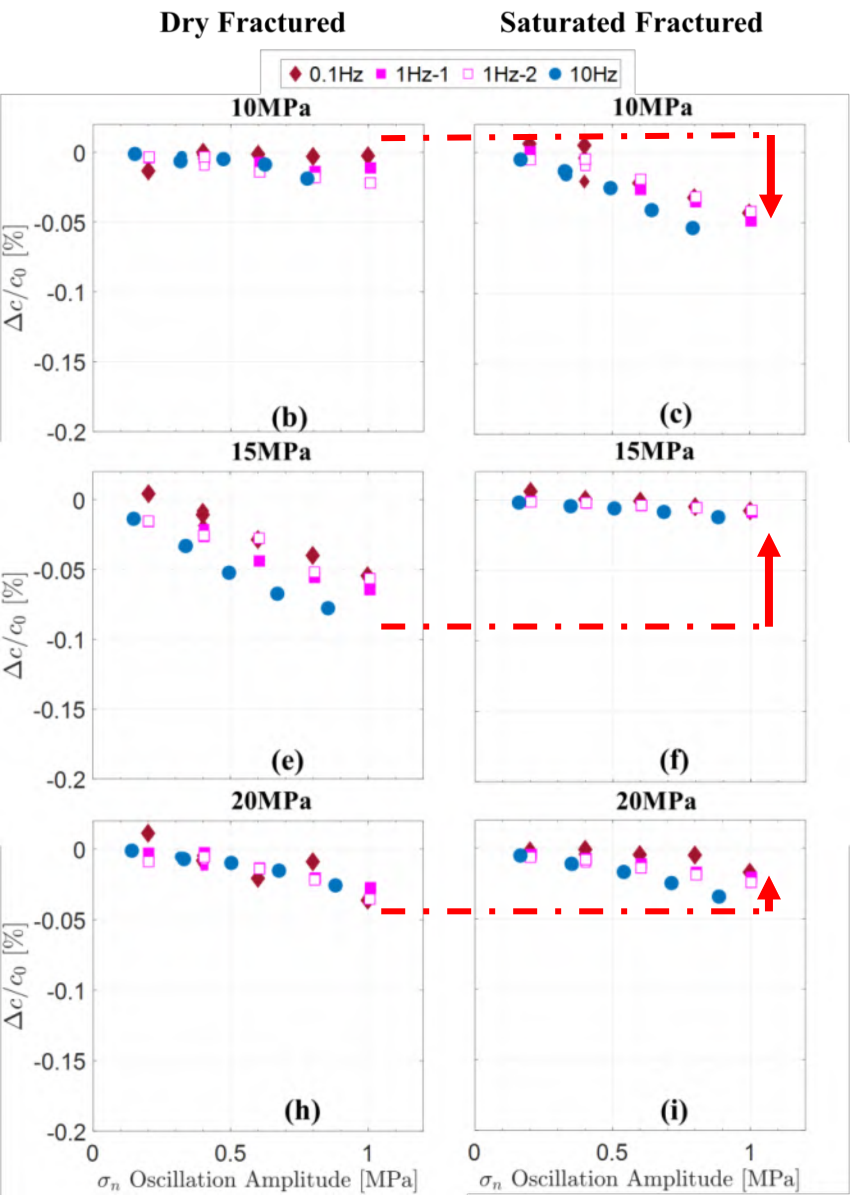


Only asperities in contact are highly strained

$$\left.\frac{\Delta c}{c_0}\right|_{\substack{intact\\sample}} \approx O_i + \beta_i \epsilon_i > \left.\frac{\Delta c}{c_0}\right|_{\substack{fractured\\sample}} \approx O_i + \beta_i \epsilon_b + \left(\frac{\Delta k}{k_0}\right)_f$$

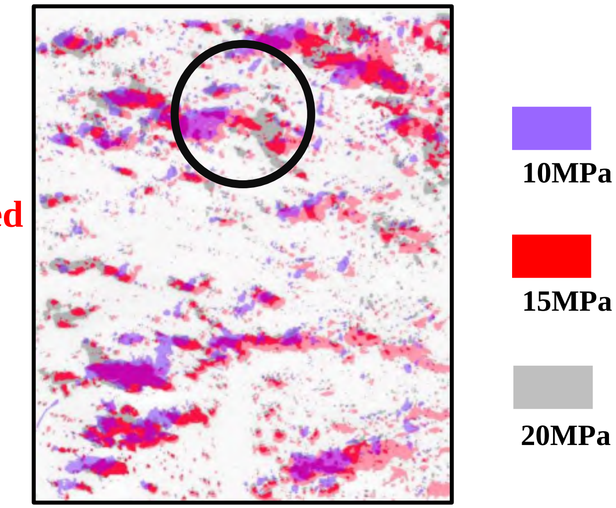
$$\epsilon_b << \epsilon_i < \epsilon_f$$

Why saturated sample exhibits less nonlinearity than dry fractured sample except at 10 MPa?

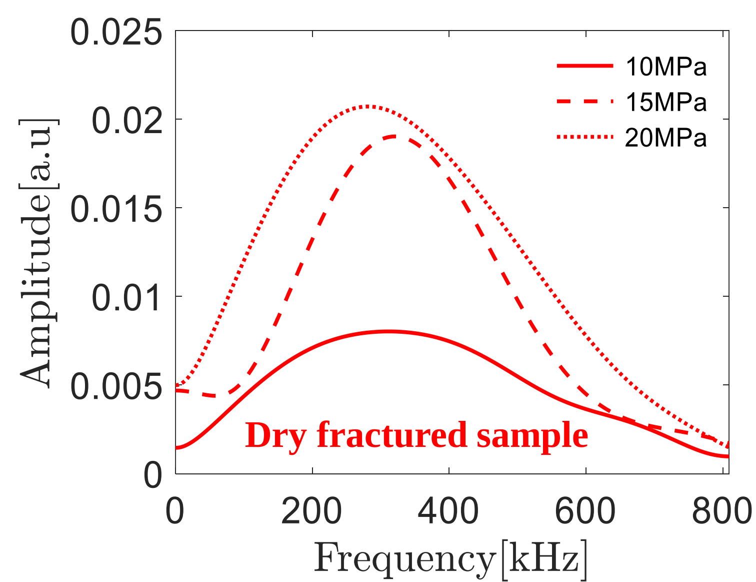


Actual contact area is less than 10% @10 MPa

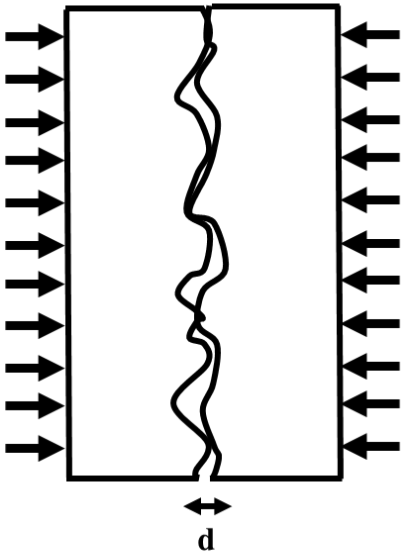
Dry fractured sample

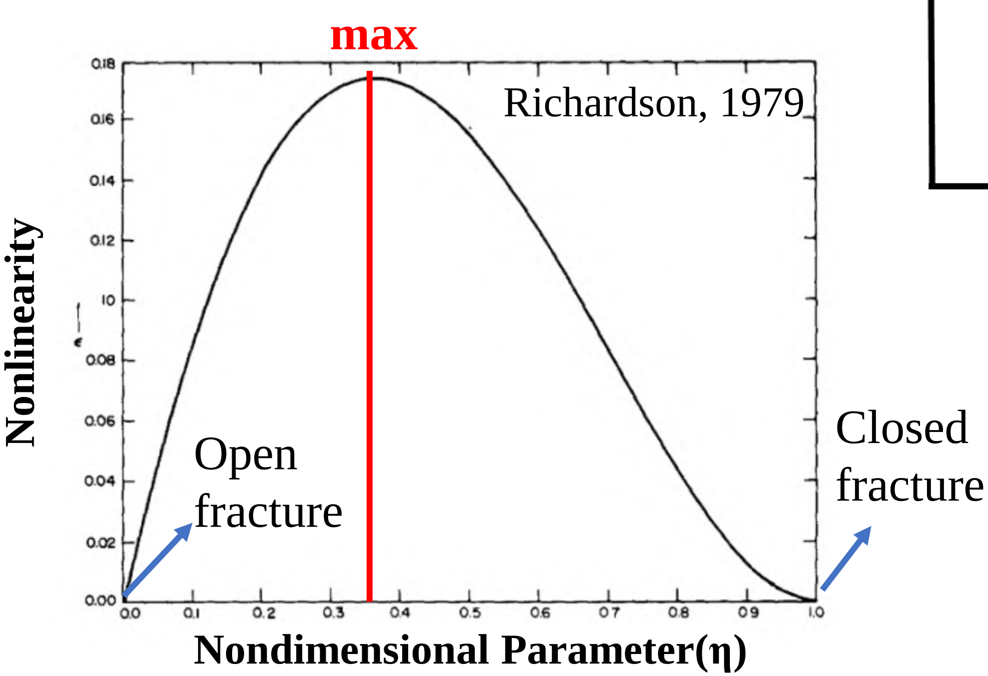


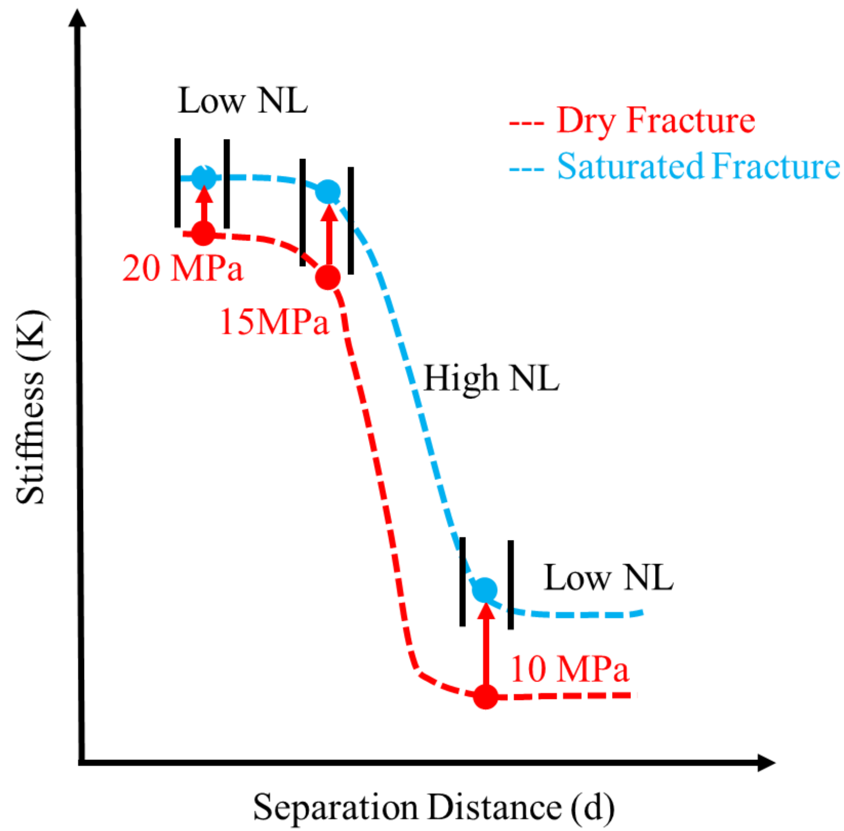
≅Transducer footprint area



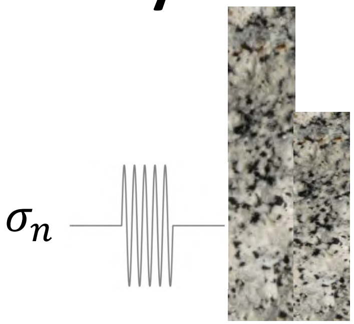
In-situ normal stress and saturation influences the measured nonlinearity



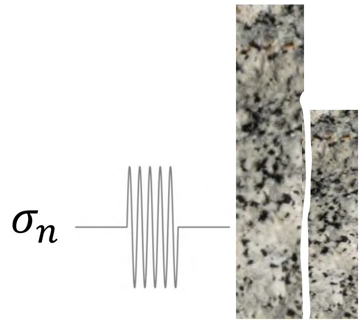




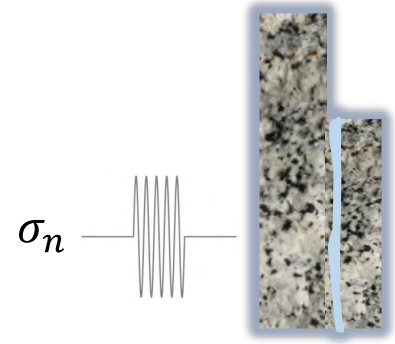
In-situ normal stress and saturation influences the measured nonlinearity



Dry intact



Dry fractured



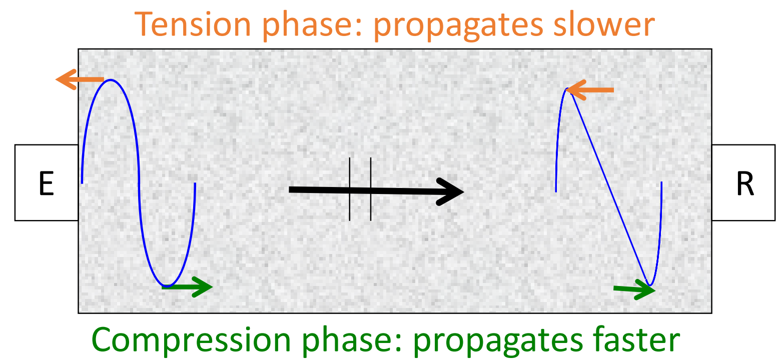
Saturated Fractured

$$\left.\frac{\Delta c}{c_0}\right|_{\substack{intact\\sample}} > \left.\frac{\Delta c}{c_0}\right|_{\substack{dry\\fractured}} > \left.\frac{\Delta c}{c_0}\right|_{\substack{saturated\\fractured}}$$

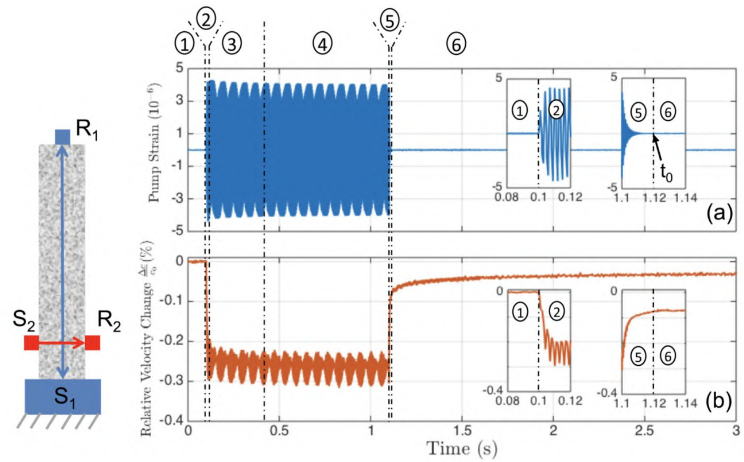
- Surprisingly, fractured sample exhibits low nonlinearity than intact sample
- Saturated sample shows less nonlinearity than the fractured sample except at 10 MPa.
- The three nonlinear parameters $\Delta c/c_0$, $\mathrm{d}c/c_0$, p_1 behaves in a similar fashion.
- Fracture aperture and contact area play important roles in the observed trends at different normal stress levels.

Wrap-Up

Principles: Nonlinear *vs* linear wave propagation; nonlinear ultrasonic testing (NRUS, SHG, DAET)



DAET: a comprehensive picture of materials nonlinearity used for intact rock characterization as well as fractures/cracks



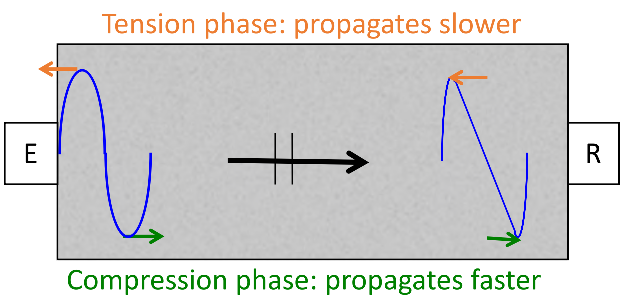
DAET on rock under in-situ conditions: Stress and saturation greatly influence the measured nonlinearity

$$\left.\frac{\Delta c}{c_0}\right|_{\substack{intact\\ sample}} > \left.\frac{\Delta c}{c_0}\right|_{\substack{dry\\ fractured}} > \left.\frac{\Delta c}{c_0}\right|_{\substack{saturated\\ fractured}}$$

Outline

Principles:

- Linear *vs* Nonlinear Wave propagation
- How to measure nonlinear elasticity?



Dynamic Acousto-Elastic Testing (DAET):

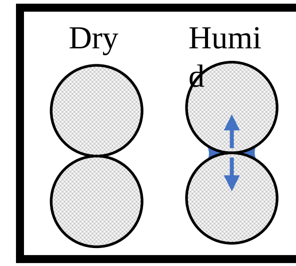
- How to do DAET?
- Intact rock characterization
- Fractured rock’s response under in-situ stress and saturation

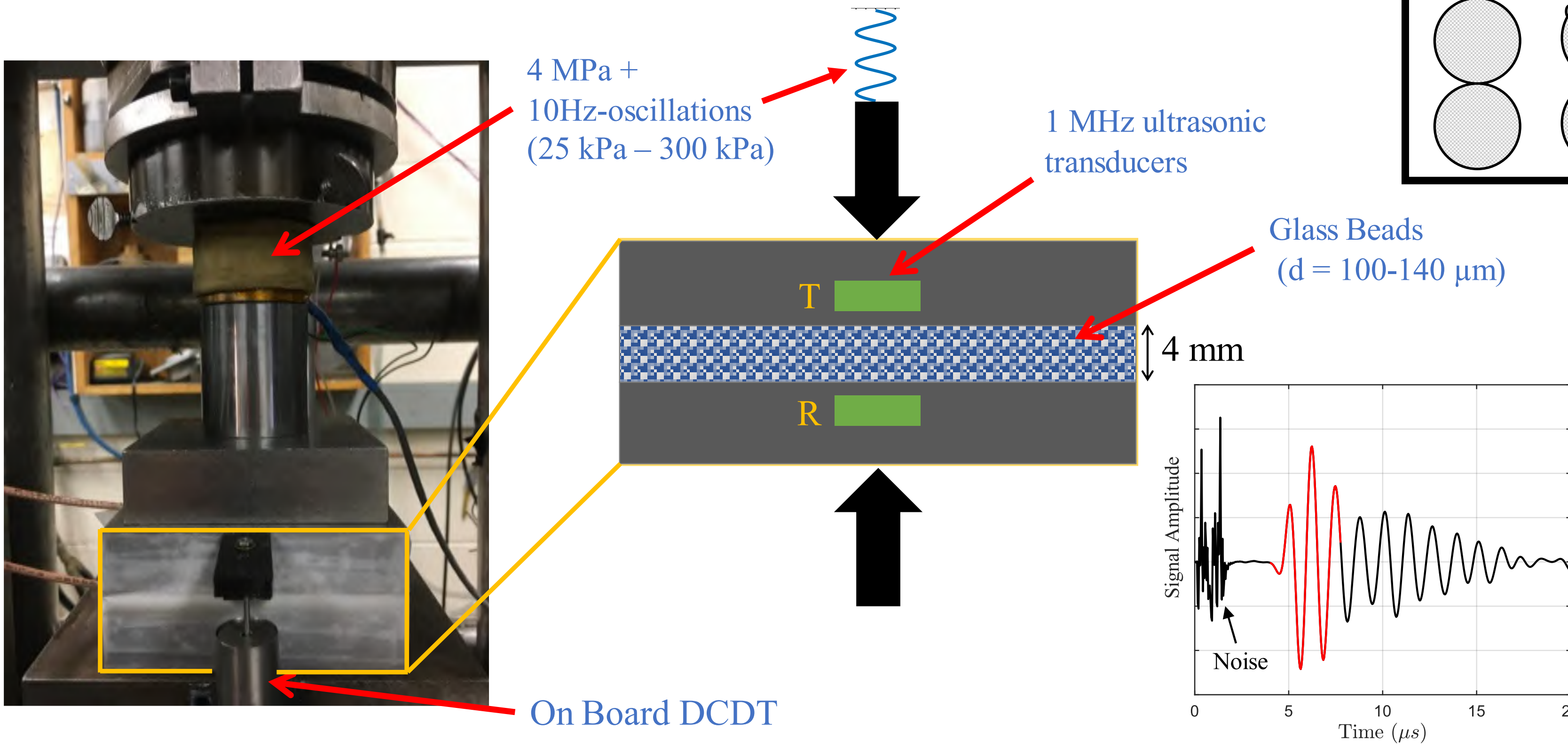
Outlook:

- New observations in granular media, Coupled X-ray CT and DAET experiments

We conduct DAET on samples of glass beads under various RH conditions

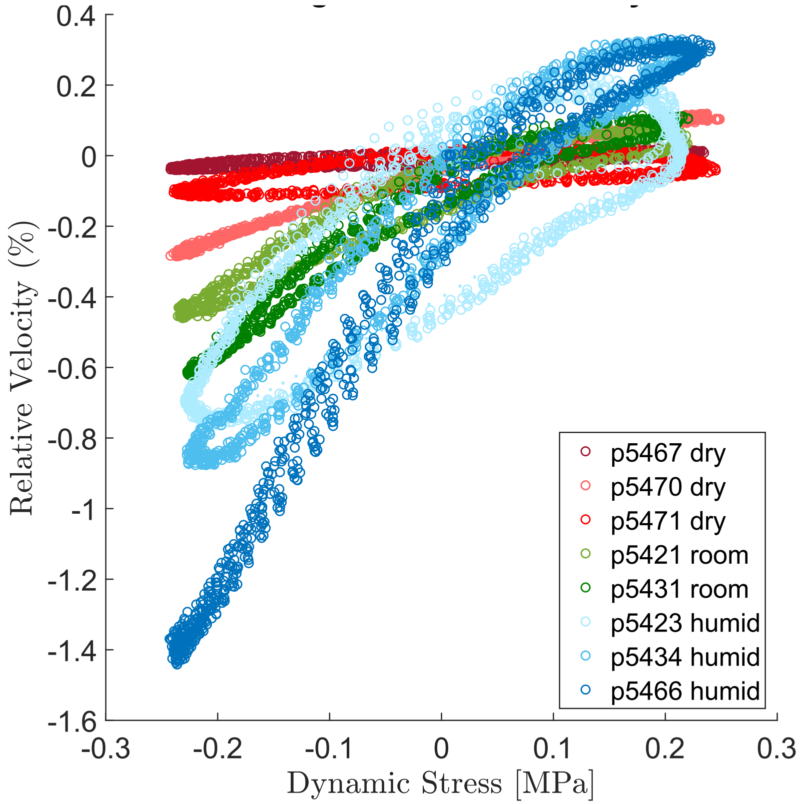
Gao, Shokouhi, Riviere, *JAP* 2022

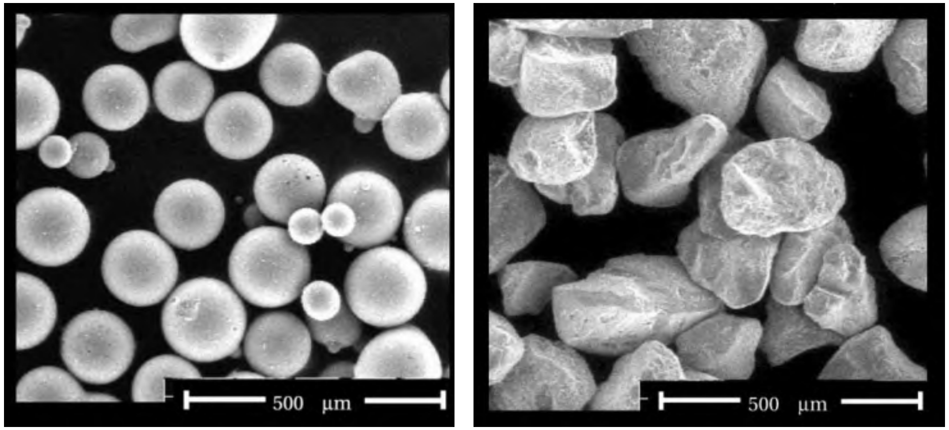




Nonlinearity in glass beads increases with RH, but not in sand ! – likely due to the differences in grain shape.

Gao, Shokouhi, Riviere, *JAP* 2022





J.L. Anthony and C. Marone, J. Geophys. Res. (2005).

Coupled time-lapse x-ray CT and DAET experiments