Nonlinear model for non-classical nonlinearity observed from lab to global scales

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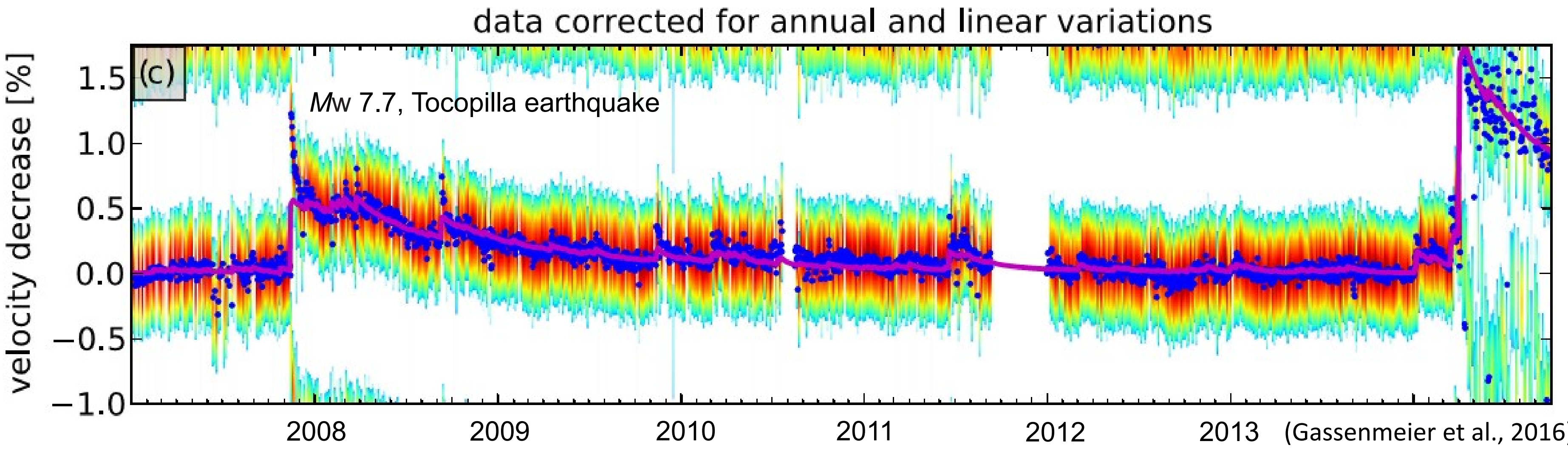
SPIN MONITORING A RESTLESS EARTH

Background

➢ Observed nonlinearity, 4-6 Hz (depth between 100-200 m), Coda Wave Interferometry



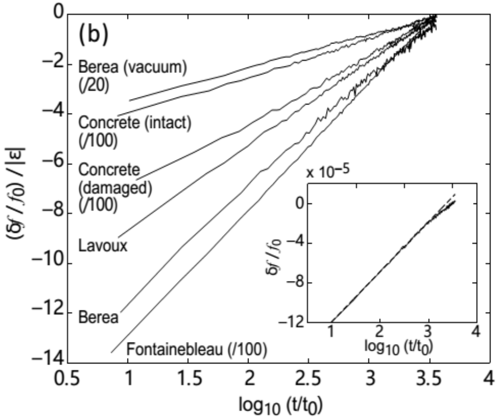
(Gassenmeier et al., 2016)

- A mathematical model
- A numerical scheme

Observation of the recovery

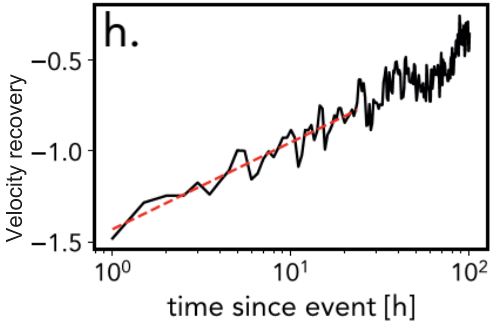
Observations

Lab



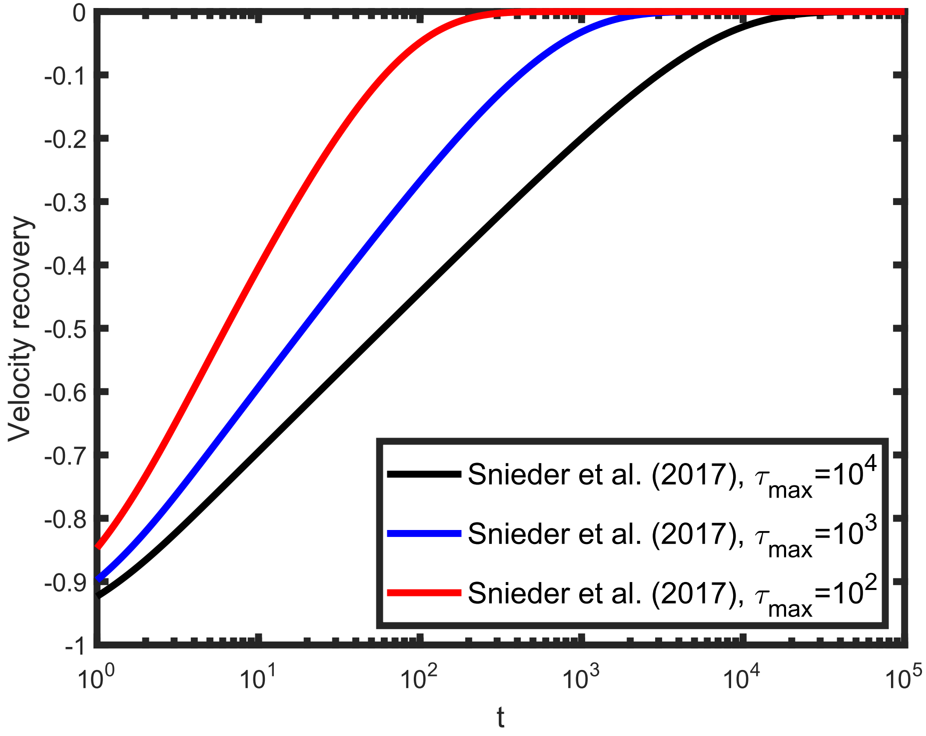
(TenCate et al., 2000)

Field

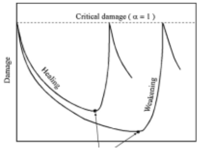


(Illien et al., 2022)

A model for recovery



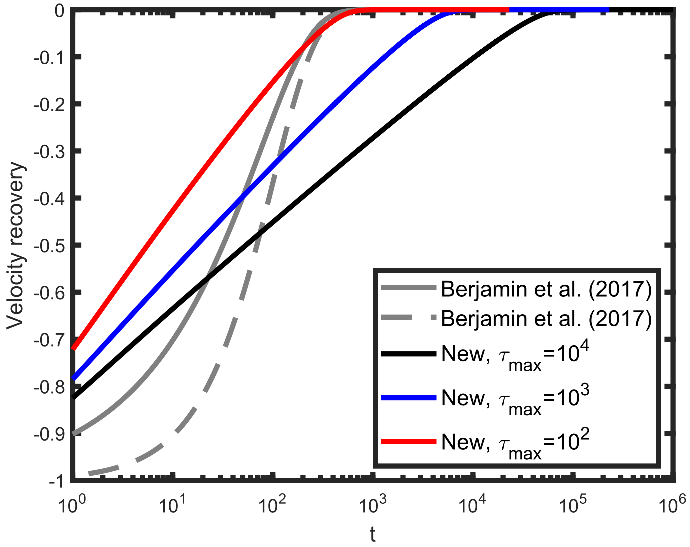
Nonlinear model selection

Models	Form	Properties	Numerical methods
Friction of internal interfaces and contact aging	$M = M_0 + \Delta M_\mathrm{l} + \Delta M_\mathrm{c} + \Delta M_\mathrm{s}$	1, 2, 3, 4	-
Internal variable Model	$\rho_0 e = \Phi_1(g)W(\chi) + \Phi_2(g)$ $\dot{g} = -\frac{1}{\tau_1}(\Phi_1'(g)W(g) + \Phi_2'(g))$	1,2,3,4	FVM
Empirical model with memory	$\frac{\Delta M}{E} = C(\tilde{t}, \varepsilon) + \beta\varepsilon + \delta\varepsilon^2$ $C(\tilde{t}, \varepsilon) = \int_0^{\tilde{t}} e^{\gamma(t-\tilde{t})} \left(\beta\varepsilon(t) + \delta\varepsilon(t)^2\right) dt$	1,2,3?	-
Continuum Damage–Breakage Faulting Model	 	1,2?,3,4, micro-cracks	FDM
Nonlinear dynamic rupture processes in diffuse fracture zones	$\partial_t\alpha + v_k\partial_k\alpha = 0, \quad \partial_t\bar{\rho} + \partial_k(\bar{\rho}v_k) = 0,$ (2.1a) $\partial_t(\bar{\rho}v_i) + \partial_k\left(\bar{\rho}v_iv_k + \alpha p\delta_{ik} - \alpha\sigma_{ik}\right) = \bar{\rho}g_i,$ (2.1b) $\partial_tA_{ik} + \partial_k(A_{im}v_m) + v_m\left(\partial_mA_{ik} - \partial_kA_{im}\right) = -\theta_1^{-1}(\tau_1)E_{A_{ik}},$ (2.1c) $\partial_tJ_k + \partial_k\left(v_mJ_m + T\right) + v_m\left(\partial_mJ_k - \partial_kJ_m\right) = -\theta_2^{-1}(\tau_2)E_{J_k},$ (2.1d) $\partial_t\xi + v_k\partial_k\xi = -\theta E_\xi,$ (2.1e) $\partial_t(\bar{\rho}S) + \partial_k\left(\bar{\rho}Sv_k + \bar{\rho}E_{J_k}\right) = \bar{\rho}\,(\alpha T)^{-1}\left(\theta_1^{-1}E_{A_{ik}}E_{A_{ik}} + \theta_2^{-1}E_{J_k}E_{J_k} + \theta E_\xi E_\xi\right) \geq 0,$ (2.1f) $\partial_t(\bar{\rho}E) + \partial_k\left(v_k\bar{\rho}E + v_i(\alpha p\delta_{ik} - \alpha\sigma_{ik}) + q_k\right) = \bar{\rho}g_iv_i,$ (2.1g)	1, 2?,4, micro-cracks	ExaHype (DG+FVM)

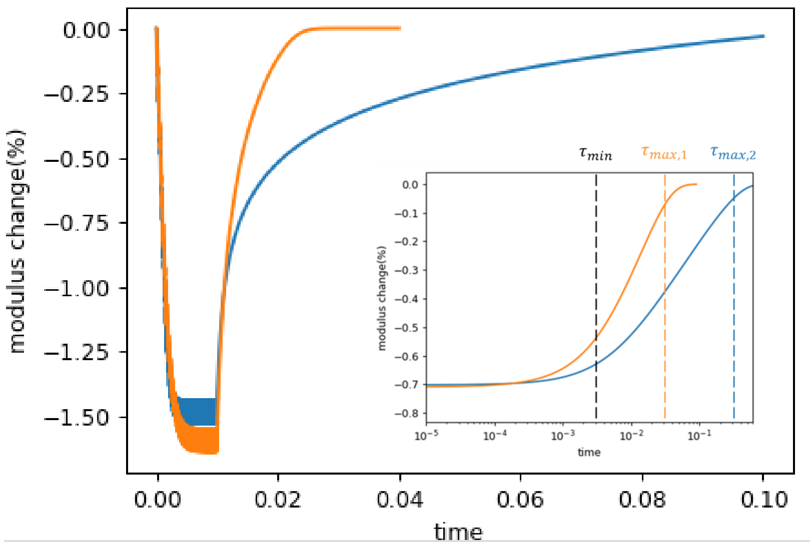
Softening[1]; Histeresis[2]; Healing[3]; Thermodynamically consistent[4]

A mathematical model for recovery in nonlinear wave equations

Solve recovery in wave equations



Conditioning and recovery



Modified Internal Variable Model[1]

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial\big((1-g)\cdot\sigma(\varepsilon)\big)}{\partial x} = 0$$

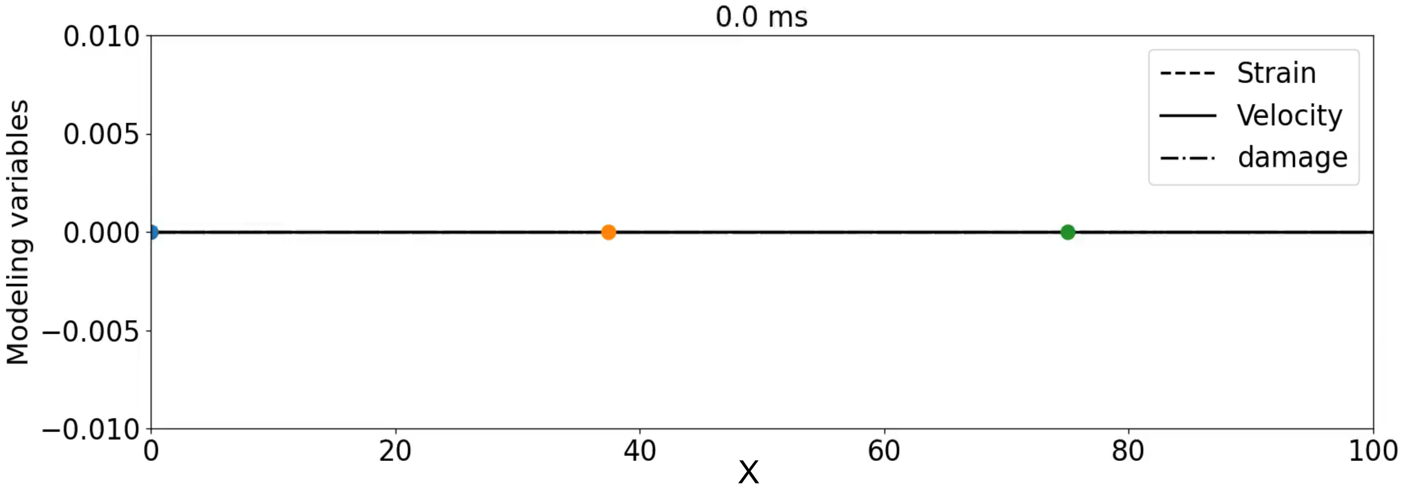
$$\frac{\partial g}{\partial t} = \left(\frac{W(\varepsilon)}{\gamma \cdot \tau_{min}} - \phi_2'(g)\right)$$

$$\sigma(\varepsilon) = E\varepsilon\big(1 + \beta\varepsilon + \delta\varepsilon^2 + o(\varepsilon^2)\big)$$

$$\phi_2'(g) = \frac{g}{\tau_{max}\left[\frac{g}{g_0}\ln\frac{\tau_{max}}{\tau_{min}}\left(1-\log\left(\frac{g}{g_0}\right)\right)+1\right]e^{-\ln\frac{\tau_{max}\,g}{\tau_{min}\,g_0}} + \tau_{min}\frac{g}{g_0}\cdot\ln\frac{\tau_{max}}{\tau_{min}}}$$

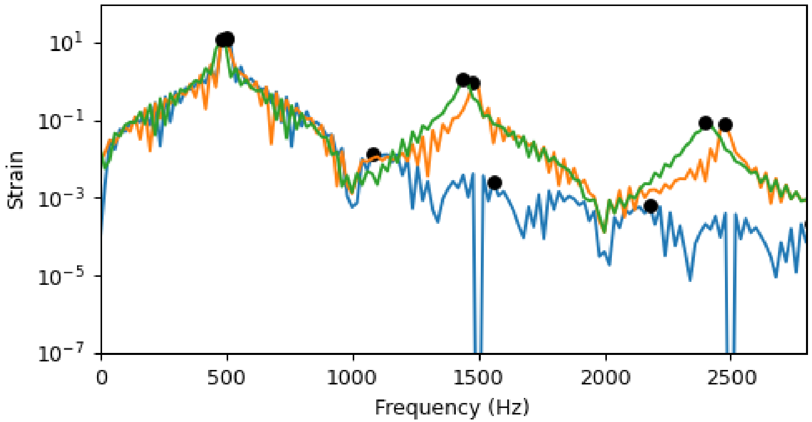
[1]Berjamin et al., (2017)

Nonlinear effects on waveform



DG method with FEniCS[1]:

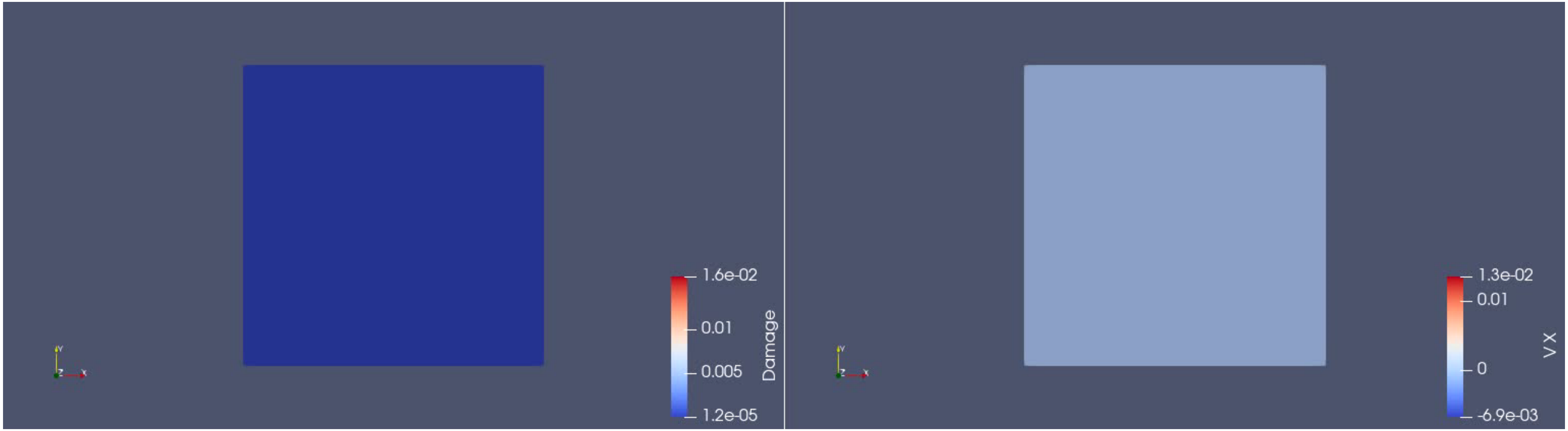
- 1st order Lagrange elements;
- 2nd order Runge-Kutta time integration;
- Roe solver for interface fluxes;
- Start from 1D



[1]Scroggs et al., (2022)

From 1D to 2D/3D

- Local Lax-Friedrichs flux for interface flux



Outlook

- Compare with experimental observation, Constrain the model parameters from experiment
  - Strain-rate (frequency) dependent behavior
  - Constrain parameters from transient (frequency) response
  - Amplitude dependent damage
  - When exponential and when logarithmic?

Rock sample

- Invert for spatial distribution of nonlinear parameters
  - Non-destructive testing
  - Use slow dynamics parameters for detection?

Structures

- Bring the simulation to the field
  - Observation of the transient effects during the damage phase of large earthquakes
  - Attenuation vs Nonlinear
  - Difference between the main shock and after shock site effect
  - Volume on Earth where nonlinearity is important/observed

Field