Non-classical elastic behavior: A continuum mechanics perspective

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Traumatic
brain injury

NUI Galway

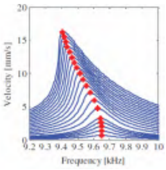
- Galway: 80 000 hab
- NUIG: 18 000 students, 2 700 staff
- Maths: 30 academics

www.esmc2022.org

Marseille, France

PhD (2015-2018)

Nonlinear acoustics

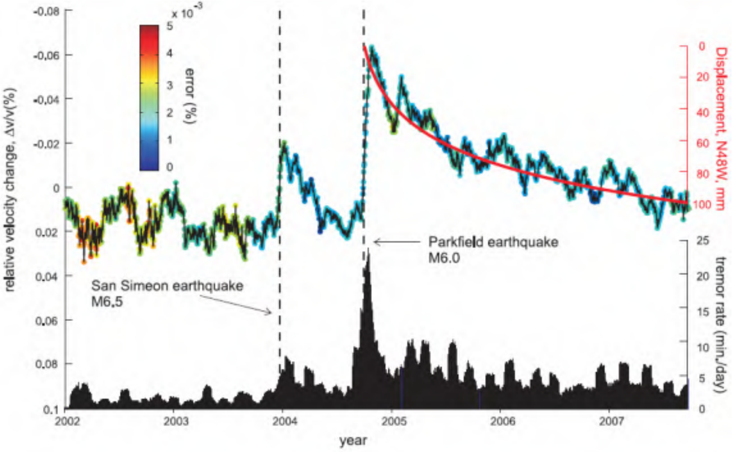


Subject

Modelling the softening behaviour of geomaterials

1. Softening behaviour of geomaterials
2. Traumatic brain injury

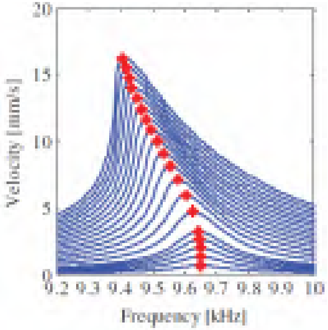
Seismology



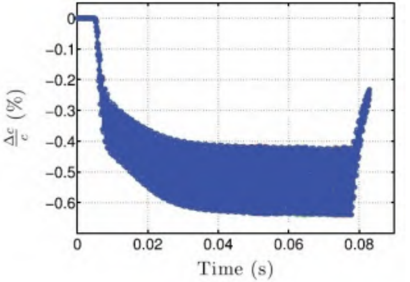
Brenguier et al. 2008

Experimental overview

Vibrating sandstone bar *TenCate et al. 2019*



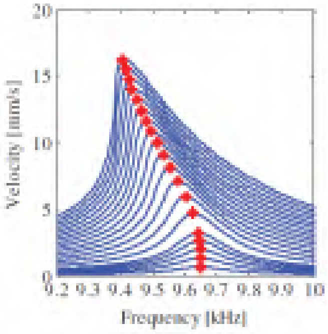
Resonance



Dynamic Acoustoelasticity (DAE)

Experimental overview

Vibrating sandstone bar *TenCate et al. 2019*



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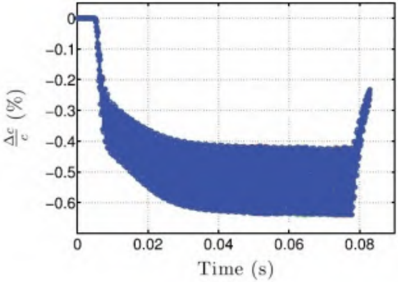
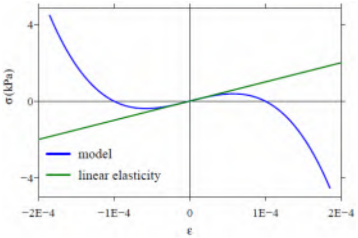


Dynamic Acoustoelasticity (DAE)

Modelling literature

Nonlinear elasticity + viscosity

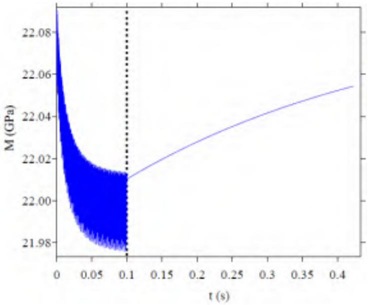
$$\sigma = \underbrace{E\left(\varepsilon + \dots\right)}_{\sigma^{\mathrm{e}}(\varepsilon)} + \eta\dot{\varepsilon}$$



Soft-ratchet model *Vakhnenko et al. 2004*

$$\sigma = (1 - g)\,\sigma^{\mathrm{e}}(\varepsilon) + \eta\dot{\varepsilon}$$
$$\dot{g} = \omega_{\mathrm{r}}\left(g_{\mathrm{eq}}(\sigma) - g\right)$$

Simulation *Favrie et al. 2015*

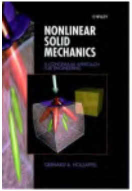


Solid mechanics

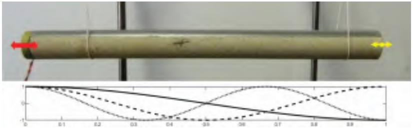
Damage mechanics

Nonlinear viscoelasticity

- nonlinear behaviour
- transients, dissipation
- reversible, repeatable



Continuum assumption



Classical mechanics

Kinematics · Balance laws · Thermodynamics

Clausius–Duhem inequality

$$\mathscr{D} = \left(\sigma - \frac{\partial U}{\partial \varepsilon}\right)\dot{\varepsilon} - \frac{\partial U}{\partial g}\dot{g} \geq 0$$

Nonlinear elasticity + viscosity

✓ consistent: $\mathscr{D} = \eta\dot{\varepsilon}^2 \geq 0$

$$\sigma = \overbrace{\sigma^{\mathrm{e}}(\varepsilon)}^{\partial U/\partial \varepsilon} + \eta\dot{\varepsilon}$$

Soft-ratchet model

✗ not checked initially

✗ not satisfied *Berjamin et al. 2017*

$$\sigma = \overbrace{(1-g)\,\sigma^{\mathrm{e}}(\varepsilon)}^{\partial U/\partial \varepsilon} + \eta\dot{\varepsilon}$$

$$\dot{g} = \omega_{\mathrm{r}}\left(g_{\mathrm{eq}}(\sigma) - g\right)$$

Internal-variable model

Keep the stress *Berjamin et al. 2017*

$$\sigma = \overbrace{(1 - g)\, \sigma^{\mathrm{e}}(\varepsilon)}^{\partial U/\partial \varepsilon} + \eta \dot{\varepsilon} \qquad \Longrightarrow \qquad U = (1 - g)\, W^{\mathrm{e}}(\varepsilon) + \phi(g)$$

Thermodynamics: $-\dfrac{\partial U}{\partial g}\dot{g} \geq 0$

Relaxation parameter $\Omega_{\mathrm{r}} > 0$

$$\dot{g} = -\Omega_{\mathrm{r}} \frac{\partial U}{\partial g}, \qquad \text{i.e.} \qquad \boxed{\dot{g} = \Omega_{\mathrm{r}} \left(W^{\mathrm{e}}(\varepsilon) - \phi'(g) \right)}$$

Finite volume implementation

Balance laws for $\boldsymbol{q} = (\varepsilon, v, g)^\top$

$$\partial_t \boldsymbol{q} + \partial_x \boldsymbol{f}(\boldsymbol{q}) = \boldsymbol{r}(\boldsymbol{q}) + \boldsymbol{D}\,\partial_{xx}\boldsymbol{q}$$

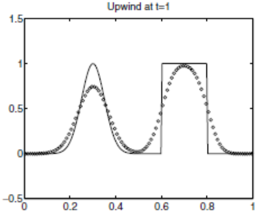
Scheme

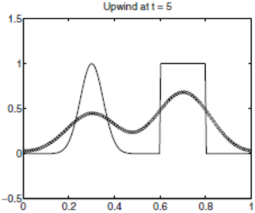
$$\boldsymbol{q}_i^* = \boldsymbol{q}_i^n + \Delta t\,(\boldsymbol{r}(\boldsymbol{q}_i^n) + \boldsymbol{D}\,\delta_{xx}\boldsymbol{q}_i^n)$$

$$\boldsymbol{q}_i^{n+1} = \boldsymbol{q}_i^* - \frac{\Delta t}{\Delta x}\left(\boldsymbol{f}_{i+1/2}^* - \boldsymbol{f}_{i-1/2}^*\right)$$

Why finite volumes? *LeVeque 2002*

$$q_t + q_x = 0$$





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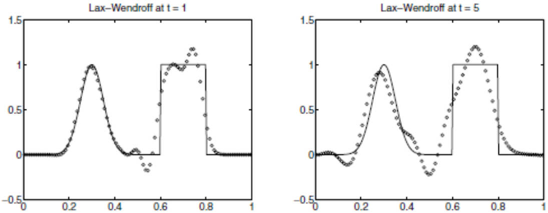

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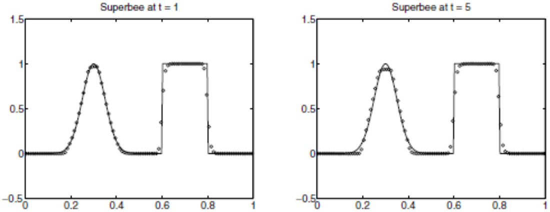
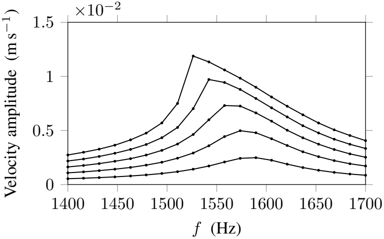
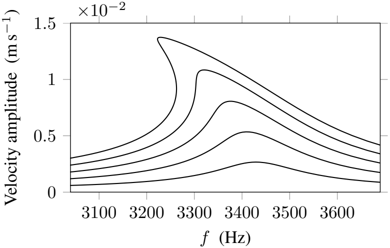


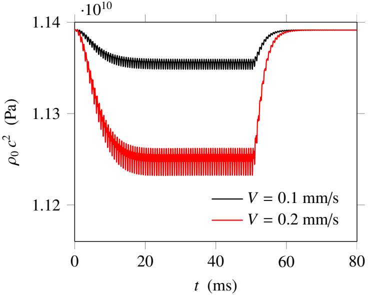
Results

Resonance *Berjamin et al. 2018*





DAE



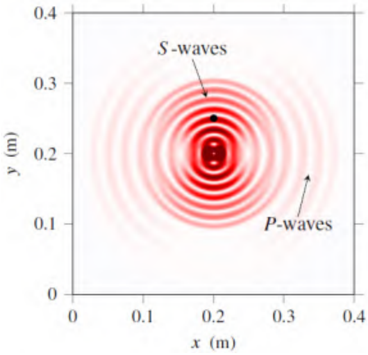
- ✓ qualitative agreement
- ✓ parametric studies

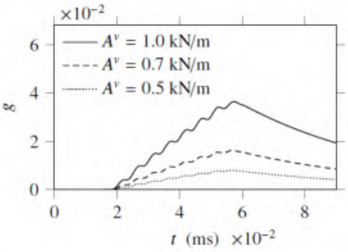
Multiple space dimensions

2D-3D *Berjamin et al. 2019*

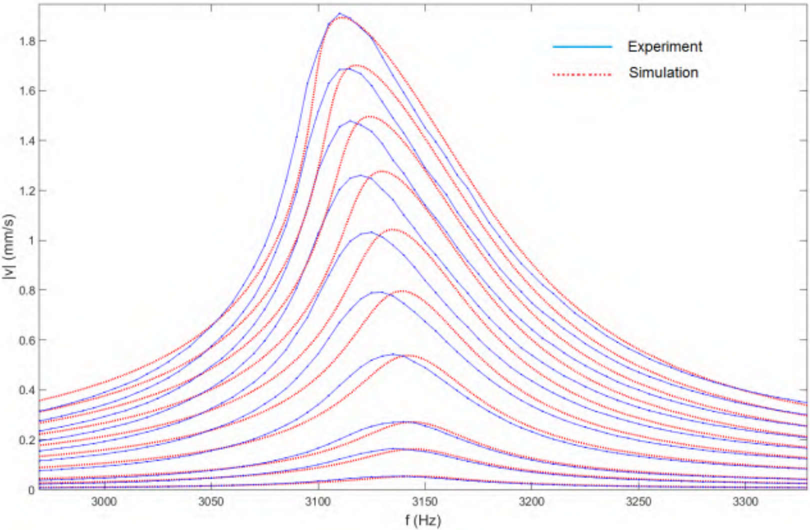
$$\sigma = (1 - g)\,\sigma^{e}(\varepsilon) + \eta\dot{\varepsilon} \qquad \Longrightarrow \qquad \boldsymbol{S} = (1 - g)\,\boldsymbol{S}^{e}(\boldsymbol{E}) + \zeta(\mathrm{tr}\dot{\boldsymbol{E}})\boldsymbol{I} + \eta\dot{\boldsymbol{E}}$$

$$\dot{g} = \Omega_r\left(W^{e}(\varepsilon) - \phi'(g)\right) \qquad \qquad \dot{g} = \Omega_r\left(W^{e}(\boldsymbol{E}) - \phi'(g)\right)$$





Quantitative agreement?



Conclusion

Change of material properties?

Micromechanics: Damage? Hydraulics? Friction?

Sensitivity with laboratory conditions

Complex rheological behaviour

Biomechanics

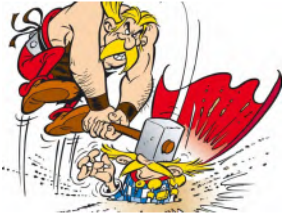
Mechanical waves

- imaging, testing
- nonlinear (visco)-elasticity

Soft tissues

- nearly incompressible ($\kappa \simeq 10^3 \mu$)
- multiphasic

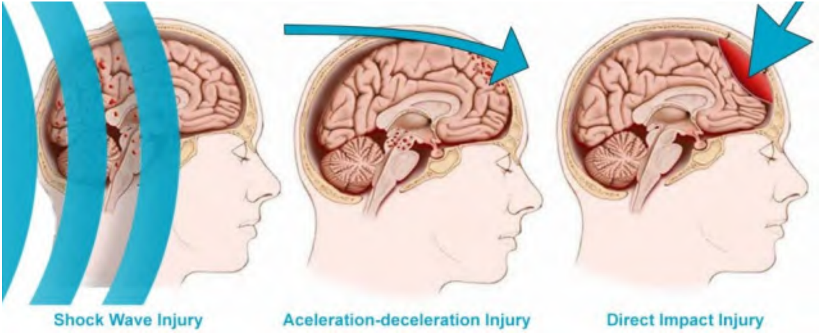
Traumatic brain injury



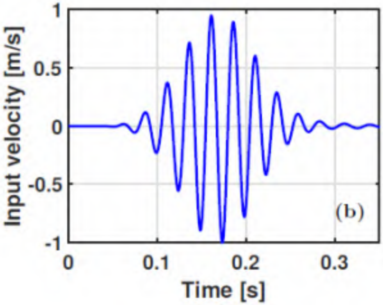
Primary injury: motion

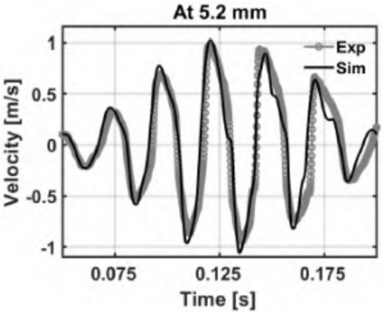
Secondary injury: swelling

Primary injury



Shock wave formation





Chandrasekaran et al. 2021

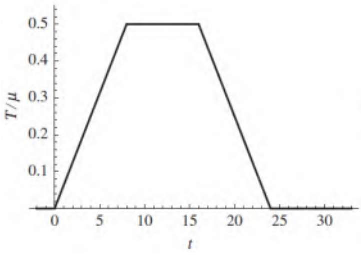
Soft viscoelastic tissue

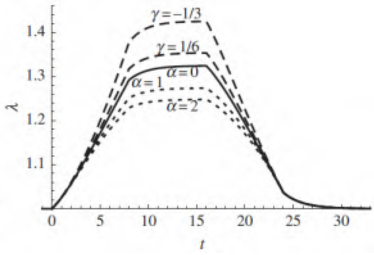
Quasi-linear viscoelasticity *De Pascalis et al. 2014*

$$\boldsymbol{S} = -p\boldsymbol{C}^{-1} + \mathscr{G} * \dot{\boldsymbol{S}}^{\mathrm{e}}$$

Prony series

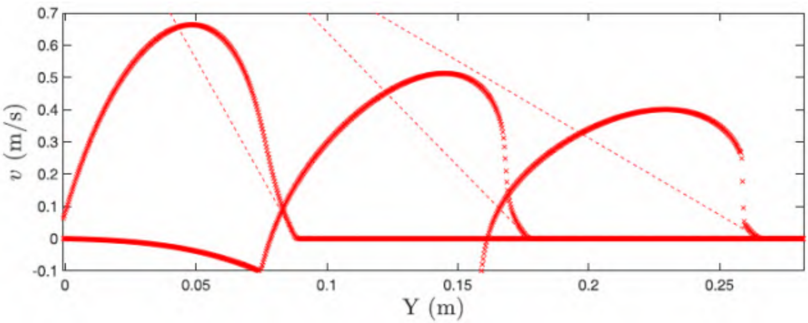
$$\mathscr{G}(t) = \left[1 - g\left(1 - \mathrm{e}^{-\omega_{\mathrm{r}} t}\right)\right] \mathrm{H}(t)$$





Direct impact simulation

Shock formation distance *Berjamin et al. 2022*



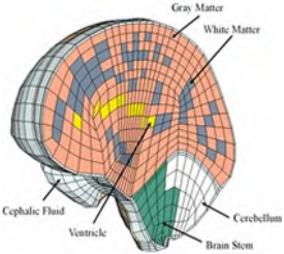
✗ approx. estimations fail

Available soon

Poroelastic behaviour

Swelling model

3D code



Acknowledgements

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Thank you for your attention!

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